

Chapter 3

The Aesthetic of Sensibility**§ 1. The Synthesis in Sensibility**

The synthesis in sensibility is the process leading to apprehension in consciousness and has for its outcome the phenomenon of perception. However, human beings have no conscious experience of the acts of this synthetical process and so our treatment of the aesthetic of sensibility must necessarily belong to Slepian's facet B of our theoretical understanding. Of the representations belonging to and in the synthesis of sensibility, only two types of representations enter into experience and are thereby principal quantities of facet B: empirical intuitions and affective perceptions. Furthermore, the Objects of facet A for which these are principal quantities are psychological Objects drawing their objective validity from the Organized Being's experience of its own intelligible Nature.

The other constituents of the theory of aesthetic in sensibility, illustrated in Figure 3.1.1 below, are entirely Slepian secondary quantities. Their *Dasein* is of intrinsically mathematical origin. They owe their place in the theory of aesthetic to the deduction of logically necessary conditions and operations required for the presentation of affective perception and empirical intuition as human beings experience these. Figure 3.1.1 thus illustrates the logical structure of the process of the synthesis in sensibility. This section provides a general overview of sensibility and the sections that follow discuss in more depth the specific acts of synthesis in our logical division of this quite primal capacity of *nous*.

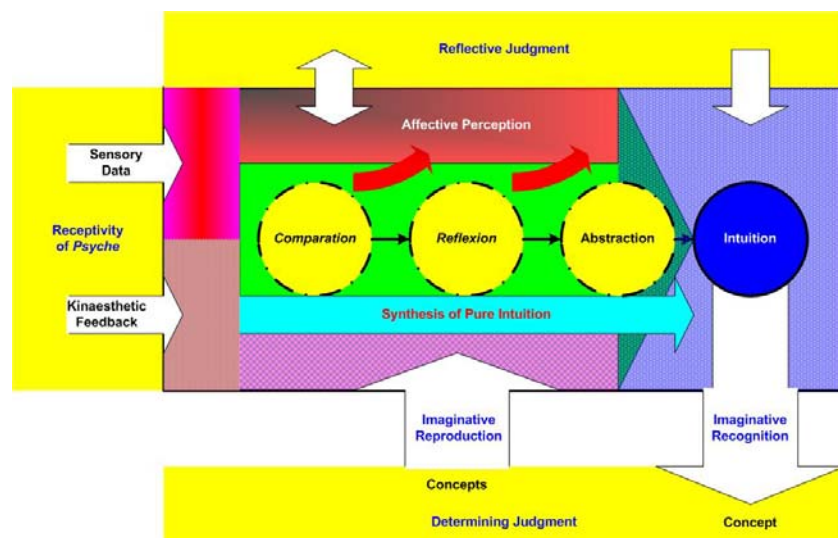


Figure 3.1.1: Illustration of the process of synthesis in sensibility and interconnected processes.

§ 1.1 The *Materia ex Qua* and *Materia Circa Quam* of Sensibility

Information enters the synthesis of sensibility from three distinguishable sources, two via *psyche's* power of receptivity and one from the spontaneity of the Organized Being by means of the synthesis of reproductive imagination in *nous*. These sources are labeled sensory data, kinaesthetic feedback, and concepts in Figure 3.1.1. Collectively, we will call these sources the matter *of* sense or ***materia sensus***. The representation of this matter *in* sensibility will be called the matter *for* sensations or ***materia sensibus***. The *materia sensibus* is represented in Figure 3.1.1 by the colored regions at the far left side and bottom left sides of the sensibility process where the entry arrows from receptivity and imagination flow into the process.

We interpret *materia sensus* as referring to the transcendental source of the information that is re-presented as the input data (*materia sensibus*) to the synthesis in sensibility. For the *materia sensus* of sensory data and kinaesthetic feedback this transcendental source is laid to sensory signaling representations in *soma*. This is to say these matters of sense refer to signals in the nervous and endocrine systems of the body representing the traditional five senses (seeing, hearing, touching, tasting, smelling) as well as other distinguishable senses (e.g. vestibular senses, skeletal muscle and joint sensory nerves, and other afferent nervous system signaling processes). These somatic sources are those traditionally regarded as making up the body's sensory systems properly so called. The other source of *materia sensus* is provided by concepts and, thus, its transcendental place lies within the division of *nous*. You will recall that a concept is a rule for the reproduction of an intuition.

Mathematically, receptivity and the synthesis of reproductive imagination are to be regarded as information-preserving transformations that re-present the source data as matter *in* the synthesis of sensibility. The ideas of "information" and "data" are quite distinct in our theory, a distinction that is fundamental in the mathematical science of information theory (cf. [WELL2]). A datum is a *representation of* something, and this something that is represented is what we call information. For example, the number "three" is represented in Arabic numerals by the symbol "3" and in digital computer storage by the packet of "bits" 00000011. These are *data representations* and they both represent the concept of the number "three." Formally, **information** is that which is persistent from one data representation to another. As objects, somatic signals and noetic concepts are representations that are quite non-homogeneous with the mathematical function of modeling sensibility. Receptivity and imagination can thus both be regarded as **schemata of re-presentation** producing the homogeneous *materia sensibus* of sensual representation. Their function is analogous to the function performed by a voltmeter in converting an electric potential difference (a physical signal) into a meter reading, e.g. "3.12

volts," (another and quite different physical signal). Here the electric potential difference is analogous to *materia sensus* and "3.12 volts" is analogous to *materia sensibus*. The voltmeter transforms an electric field matter into a visible matter (the meter display).

The representation of *materia sensibus* in sensibility is not a conscious representation. In technical terminology, unconscious representations are called **obscure representations**. All obscure representations in mental physics are Slepian secondary quantities. This is because an obscure representation cannot be set in an immediate relationship to experience and experience is the ground of objective validity for all empirical concepts and ideas *and* the point of overlap between Slepian facets A and B. *Materia sensibus* is unreal in an ontological context but real in the mathematical context of theory.¹

The *functional nature* of the *materia sensibus* is further classified according to its relationship to perceptions. This relationship is either as *materia ex qua* (matter out of which) or as *materia circa quam* (matter around which) for the composition of conscious representations. We call the matter *contained in* an intuition **sensation**. The matter contained in an affective perception is called **feeling**. Sensation and feeling are both instances of the *materia in qua* (matter in which) of perception. *Materia ex qua* is the determinable matter for a determination (a determination of perception in sensibility in the case of our current discussion); *materia circa quam* is matter participating in the act of determination by which something *is given a form* (form of perception in our present context). *Materia in qua* is the determined matter, i.e. matter as the subject of inherence for an Object. In terms of the general ideas of Modality in representation, these are the ideas of the determinable, the determining factor, and the determination, respectively.²

§ 1.2 The Forms of Sensibility

All representations require a matter of representation (the "what" of representation) and a form of representation (the "how" of representation). The forms of affective perceptions and empirical intuitions are quite different in kind. Affective perception is a conscious representation that can never be part of the representation of an object. Affective perception is subjective, empirical intuition is objective.

In terms of a 1LAR, form is the connection of matters of composition in a manifold of representation. In the context of an Object of representation, that to which the representation refers is called the object when the representation is an intuition. We use a different term than

¹ Only a principal quantity can be real in an ontological context, and here its reality holds only insofar as its concept stands in a transcendental relationship with objects of facet A.

² Recall that the ideas of Modality pertain to meanings. We are presently discussing the meaning of the *materia* of sensibility.

"object" when speaking of affective perception because in this case the object is the Organized Being itself regarded as intelligent being (Figure 1.2.4). We refer affective perception to the Organized Being by replacing the word "object" with the term **transcendental Subject**. The transcendental Subject is the *noumenon* for which the Self is the appearance. Similarly, the direct object to which objective representation refers is called the **transcendental object**.³ It is this difference in what is referred to by the combination in representation that leads to the difference in kind between the form of empirical intuition and the form of affective perception.

Because affective perception refers only to the transcendental Subject, the form of affective perception is a representation synthesized by the process of *aesthetical* reflective judgment.⁴ This interplay between reflective judgment and sensibility is depicted in Figure 3.1.1 by the double-headed arrow linking sensibility and reflective judgment. Form of affective perception is the subjective *nexus* between sensibility and transcendental Subject and the terms we will be using to explain this connection include 'desire', 'sense of x' (where x will denote the particular kind of sense in functional terms), 'satisfaction and dissatisfaction,' and 'feeling of *Lust* or *Unlust*.' The synthesis of the form of affective perception belongs to reflective judgment and we will discuss this synthesis in a later chapter.

As for empirical intuition, it is the peculiar nature of human cognition that all our cognitions are presented in appearances of extended *things*. A **thing** is an object represented and understood in terms of it being possible to regard this object as having an actual or necessary *Existenz* independent of the Organized Being who represents that object. A thing is therefore also called an *external* object and the form of its representation is therefore called **the form of outer sense**. This applies even to our understanding of *soma* (the Organized Being as appearance in facet A; refer to Figure 1.2.4). This is one reason why *soma* in Figure 1.2.3 is depicted as the "outer ring" of the Organized Being in the model. The synthesis of the form of outer sense in Figure 3.1.1 is called **the pure intuition of space** and it is part of the process called the **synthesis of pure intuition** in that figure.

As we should anticipate from our earlier discussion of synthesis, these two forms in sensible perceptions are not sufficient to provide the overall representation of form in sensibility. The distinction between affective perception and intuition is merely a logical and practical division of these secondary quantities. A third synthesis of form is necessary to unite both in the

³ Recall that the adjective "transcendental" means "necessary for the possibility of experience." The adjective "transcendent" means "surpassing the possibility of actual experience." The idea of a transcendent object lacks any objective validity whatsoever.

⁴ In the 1LAR of reflective judgment, aesthetical reflective judgment is the matter of reflective judgment and teleological reflective judgment is the form of reflective judgment overall. We will see later that reflective judgment overall is represented by a 3LAR.

consciousness of the Organized Being. This third form required by our theory is called **the form of inner sense** and its synthesis is called **the pure intuition of time**. This synthesis, like the pure intuition of space, is part of the overall synthesis of pure intuition in Figure 3.1.1. The form of inner sense performs the functional role of providing connection between the *nexus* of affective perceptions and the *nexus* of empirical intuitions.

Treatment of the synthesis of pure intuition belongs to the Aesthetic of sensibility. Therefore, we will treat both the form of outer sense, with its pure intuition of space, and the form of inner sense, with its pure intuition of time, in this chapter.

§ 1.3 The Logical Synthesis of the *Verstandes-Actus*

One can say without doing injury to our understanding that the *materia sensibus* is amorphous ("without form"; the connotation is like that of a gas vs. a solid). The synthesis in sensibility takes this amorphous matter and from it produces objective representations (empirical intuitions) having both determined matter (*materia in qua*) and form. In the logical classification of our secondary quantities, the *materia in qua* of both affective perception and intuition is regarded as being determined *from* the *materia ex qua* of sensory data. The form of the pure intuition of space, in contrast and as will be explained in the section on the transcendental Aesthetic of space, is logically regarded as being formed from the *materia circa quam* of kinaesthetic feedback. (This is why the *materia sensus* is logically divided between sensory data and kinaesthetic feedback and why the latter is called the matter *around which* after the transformation of receptivity to *materia sensibus*). The synthesis of *materia ex qua* in the determination of sensation and feeling in perception is, logically, a three-step process (because synthesis is inherently three-fold) called the *Verstandes-Actus* or "acts of understanding."

These acts follow a logical progression in which the logical steps are named **Comparison**, **Reflexion**, and **Abstraction**. In this sub-section the logical product of each step is explained. More detailed discussion will be provided in section 4 of this Chapter. The present discussion is limited to the particular case of the synthesis of intuition; we will deal with affective perception in the more detailed discussion later.

The term *Comparison* is an obsolete English word, used as a technical term by Kant, from the Latin root *comparatio* (comparing; a putting together; a weighing of relative values and merits [of]). *Comparison* carries the connotation of being a merely logical comparison of a multiplicity, namely the *materia ex qua* of our *materia sensibus*. In this context, we can regard this *materia ex qua* as comparates (things being compared). Kant described *Comparison* as "the comparison of representations to one another in relationship to the unity of consciousness" [KANT2: 9 (94)].

This technical usage is not a particularly common connotation English speakers use for the word "comparison" since, in addition to the representations said to be undergoing comparison, it introduces the third factor of "their relationship to the unity of consciousness." Since *Comparison* is an act of synthesis (rather than an analytical function), we should be anticipating a third factor is involved; what connotation does this third factor bring with it?

Perhaps the best way to grasp this technical connotation is to go back, as Kant almost undoubtedly did, to the first Latin root of the word. This is the verb *comparo*, which has the following definitions in Latin:

1. (a) to place together or in corresponding positions; align. (b) to match, couple, unite, pair (with).
2. to match, pit, set (against).
3. to treat (one person or thing) as equal to another, put in the same class with, regard as comparable.
4. To estimate or evaluate in relationship (to each other or another person or thing).

These are the connotations that go with the act of *Comparison*. We will discuss the implications of this during the more detailed discussion later.

The word *Reflexion* translates into English as "reflection" or "reflex." We retain the German word in Anglicized form, *reflexion*, in order to maintain a distinction between this *Verstandes-Actus* and reflective judgment. Kant described *reflexion* as "consideration of how different representations can be comprehended in one consciousness" [KANT2: 9 (94)]. In *Critique of Pure Reason* he writes,

Reflexion does not have to do with objects themselves, in order to acquire concepts directly from them, but is rather the state of mind in which we first prepare ourselves to find out the subjective conditions under which we can arrive at concepts. It is the consciousness of the relationship of given representations to our various sources of knowledge, through which alone their relationship among themselves can be correctly determined. [KANT1: B316].

Kant contrasted *reflexion* with *Comparison* by describing the latter as "logical reflexion." *Comparison* ignores all material meaning and is concerned only with formal rules for making comparisons among representations. *Reflexion*, on the other hand, is concerned with the material origins of representations and determines rules of comparison by providing a link between representations and the mental abilities that produce and process them.

Abstraction is an act of separation or segregation, the removal from a representation of all *materia ex qua* that differ in comparison to the *materia* coalescing into the final determined representation. After the act of abstraction, what remains is consolidated in the determined representation to constitute the *materia in qua* of that representation.

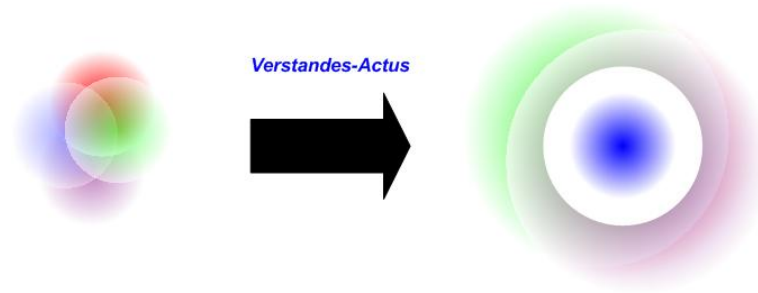


Figure 3.1.2: Synthesis of the *Verstandes-Actus* viewed as an accretion process.

We will require a more detailed explanation of *Comparison*, reflexion, and abstraction. In particular, we will need to examine much more closely this basic idea of "comparison" and have a *Realerklärung* for precisely what "comparison" (*Comparison* and reflexion jointly) is and what it does. However, it is useful as an aid to understanding to regard the outcome of the *Verstandes-Actus* as being like a kind of accretion process. Figuratively, this is illustrated in Figure 3.1.2 above. At the input to the synthesis we have the amorphous representations of the various *materia ex qua* of sensibility. During the synthesis of the *Verstandes-Actus* some of this *materia* is coalesced and combined (through *Comparison* and reflexion) to produce, for example, the *materia in qua* of an intuition. Concurrently, other *materia ex qua* is excluded (by abstraction) from this growing coalition of matter. This excluded *materia* either coalesces into another intuition, or into matters of affective perceptions, or remains obscure and unused *materia ex qua* not brought to consciousness either objectively or subjectively. It would not be misleading to say the synthesis of the *Verstandes-Actus* acts to *concentrate* the *materia* of perception.

Finally, in closing this introductory overview section, it is important to remind ourselves that there is no act of *judgment* in the synthesis in apprehension. We can and must view this synthesis as an active process set into action by the stimulation provided by the input of its *materia sensibus*, and *not* as some mere template or transcription action. But sensibility does not judge and is not the determining factor in determining what *materia* is to be an intuition, what *materia* is to be an affective perception, and what *materia* is to be excluded from attention and consciousness. The determining factor in all of this is the process of reflective judgment. The synthesis in apprehension provides, in a manner of speaking, the options available to reflective judgment but does not itself make the final determination. The role of reflective judgment in *marking* an intuition at a moment in time is represented in Figure 3.1.1 by the unidirectional arrow from reflective judgment to sensibility.

§ 2. The Transcendental Aesthetic of Space

What is 'space'? Hundreds of scholarly works have been written and published on this question

and have presented a great many different and conflicting views. In science and engineering the word 'space' is invariably used with some adjective modifier in front of it (either explicitly or implicitly). Chapter 17 of *CPPM* provides a review of the most influential of these differing views. We will not retrace that review here but we do have to at least take a brief look at the issue before presenting the Critical solution for the theory of mental physics. At the risk of trivializing the question, the various opinions regarding 'What is space?' can be roughly classified into two polar extremes we can, somewhat tongue in cheek, describe as: (1) "Space is a nothing that is a something" and (2) "Space is a something that is a nothing." Ontology-centered metaphysics, when pressed for an explanation, inevitably finds itself retreating toward one or the other of these self-contradictory positions.

Scholars, naturally, tend to be a bit more eloquent in expressing themselves than this. *The New Penguin Dictionary of Science* (2nd edition) defines "space" as

1. Any region outside the Earth's atmosphere with a very low particle density.
2. Collectively, the three dimensions that manifest themselves as distances, as opposed to the fourth dimension, time.

The first definition is really "outer space" rather than "space itself." Both definitions are ontological even though the second tries to appear to be "mathematical"; the ontology is revealed by the statement that the three dimensions "manifest themselves as distances." This is space as a thing that appears to be a no-thing.

The Oxford Dictionary of Physics provides a definition of "space" almost but not quite the same as this. Space is

1. A property of the universe that enables physical phenomena to be extended into three mutually perpendicular directions.

This definition moves the ontology from "space" to "the universe" and makes "space" a "property of the universe." But this "property" is not "just the way the universe is"; it is something "that enables" physical phenomena "to be extended." This space is a no-thing that *does* something. In the language of Critical Metaphysics this is space as a *Kraft* (power) of a substance called "the universe." The transcendent illusion here lies with the implication that "the universe" *is an entity* that "does something." Like definition 2 above, this idea utterly lacks any objective validity.⁵

Reber's *Dictionary of Psychology* lists seven definitions of "space." The primary definition, from which the others given there are derivative, is

⁵ It should be mentioned that in the new "string theories" being proposed by a number of physicists, "space" is viewed as having *more* than three dimensions. When talking down to the rest of us, string theorists tell us we do not perceive these extra dimensions because they are "small." Quite probably no more absurd statement than this could pass the lips of a serious scientist. What, sir, is a "small dimension"?

1. Fundamentally, space is an abstraction, a geometric characterization of a system of location of m objects in n dimensions. In the classic model of physical space, m is finite and $n = 3$.

Interestingly enough, this definition is closer to that given by Einstein than are either of the physics definitions given by the dictionaries above. In his 1916 paper on the general theory of relativity, Einstein wrote:

In classical mechanics, as well as in the special theory of relativity, the co-ordinates of space and time have a direct physical meaning. To say that a point-event has the X_1 co-ordinate means that the projection of the point event on axis X_1 , determined by rigid rods and in accordance with the rules of Euclidean geometry, is obtained by measuring off a given rod (the unit of length) x_1 times from the origin of the co-ordinates along the axis of X_1 . . .

This view of space and time has always been in the minds of physicists, even if, as a rule, they have been unconscious of it. . . But we will now show that we must put it aside and replace it by a more general view[.]

Einstein held that "space" and "time" were concepts that had no meaning outside the context of how positions, motion, and time were *measured* and *the rules by which these measurements are carried out*. As he elsewhere famously quipped, "Space is not a thing." The fundamental finding in the general theory of relativity was that physics must prescribe *to* mathematics the form of geometry to be used as "the rules for making measurements" and then, once so prescribed, physics must place itself under the mathematical consequences that follow. Einstein's "space and time" are *practical* ideas for defining something we here will be calling **objective space** and **objective time**. It should also be mentioned that present day relativity theorists and Big Bang cosmologists have ignored Einstein's caution about the epistemological aspects of "space" and "time" by writing and speaking of "space-time" ("space" and "time" both taken together) as if it is once again a thing with properties of its own. They have, in other words, put ontology back at the center of how they think about space-time without even being conscious that they have done so.

Mathematicians, by the nature of their work, tend to be far more precise and careful in how they define things. *The Penguin Dictionary of Mathematics* (3rd edition) does not define "space" at all. It does define something called "abstract space." Abstract space is

A set of entities, together with a set of axioms for operations on and relationships between these entities. Examples are metric spaces, topological spaces, and vector spaces.

Mathematicians avoid ontological issues altogether. We may note that their definition of abstract space is wholly a practical definition in terms of "what one does with" the notion of "space." This definition has practical objective validity within a very well delimited context.

Finally, we must not neglect the philosophers. With their post-positivism penchant for admiring the problem rather than solving it, *The Oxford Dictionary of Philosophy* merely

describes "space" in the following way:

The classical question is: is space real, or is it some kind of mental construct or artifact of our ways of perceiving and thinking? Is it 'substantial' or purely 'relative'? According to substantialism, space is an objective thing comprised of points or regions at which, or in which, things are located. Opposed to this is relationalism, according to which the only thing real about space are the spatial (and temporal) relations between physical objects.

Substantialism and relationalism are two polar extremes in the debate over the real meaning of "space"; neither of these views is Critical. We will find that the Critical answer to the question of "space" (and that of "time" as well) lies in between these extremes *as a synthesis* of these opposites. The Critical solution is, of course, epistemology-centered and ontology is subordinated to epistemology.

Here at the outset, it is crucial for us to understand that the idea of "space" must be divided by a disjunction into two pieces: **subjective space** and **objective space**. Of these two, it is subjective space that is primary and primitive. Ideas of objective space are concepts made possible and endowed with meaning only by and because of the pure intuition of subjective space. Because subjective space is primary and primitive, we will hereafter refer to it simply as "space."

§ 2.1 The *Gestalt* of Space

Space is the form of outer sense given to *materia in qua* of intuitions. In terms of metaphysics, we understand the idea of space in a two-fold fashion. Because space is not a something delivered up through receptivity (we do not sense space) nor is it a something delivered into the synthesis of apprehension by way of concepts and reproductive imagination (spatial form is necessary for the possibility of concepts and logically antecedes the representation of any concept), space is a form given to intuitions through a process of synthesis. We call this process of the synthesis of spatial form the **pure intuition of space**. It is the capacity for the formation (*Gestaltung*) of the *Gestalt* in intuition. This is the *practical* objectively valid idea of space and so space *qua* space is a *transcendental* ideal. Because the judgment that a representation in sensibility is to be marked as an intuition belongs to the process of reflective judgment, the proper Standpoint from which to understand the epistemology of space is the judicial Standpoint.

However, as a pure form of intuition, space is also primitively embedded in all human intuitions and, consequently, in all human concepts and, therefore, in all human understanding of objects. Let us recall that "to be a real object" requires that the Organized Being has a concept of the object (in which is contained the notion of matter, the real of sensation) and this concept must be connected to other concepts that give it a context and provide for its meaning. This is not possible without the representation of spatial form in intuition and so from an *empirical perspective* space is also *empirically real*. As Kant put it,

Our expositions accordingly teach us the *reality* (i.e. the objective validity) of space in regard to everything that can come before us externally as an object, but at the same time the *ideality* of space in regard to things when they are considered in themselves through reason, i.e., without taking account of the constitution of our sensibility. We therefore assert the *empirical reality* of space (with respect to all possible outer experience), though to be sure its *transcendental ideality*, i.e. that it is nothing as soon as we leave aside the condition of the possibility of all experience and take it as something that grounds the things regarded as they are in themselves. [KANT1: B44]

A transcendental idealist is also and at the same time an empirical realist.

This is the crux of the Critical (epistemological) resolution of the "What is space?" controversies noted above. Space is *both* empirically real as a thing *and* unreal as a thing, its empirical reality (objective validity) being a matter of context. It is both empirically real as a "relation" *and* unreal as a "relation." This synthesis of opposites is possible for epistemology-centered metaphysics and impossible for ontology-centered metaphysics. Let us recall that we can regard the judicial Standpoint as the synthesis of the theoretical and practical Standpoints. We can obtain a theoretical understanding of the "nature" of space, but only if this theoretical understanding is *conditioned by* the practical Standpoint of "what the pure intuition of space does."

The logical essence of our theoretical understanding of space is mathematical and operational. Einstein was right to point this out in his relativity theory, and present-day physicists are wrong to attribute properties to *objective* space as a thing-regarded-as-it-is-in-itself without regard to any accounting for the conditions of the possibility of experience as human beings know experience. Merely because our understanding of space is necessarily mathematical, this does not mean objective space is Platonic or that the emerging new ontology-centered Platonism in physics can claim any objective validity whatsoever. Pure mathematics is not about natural objects *per se*.

Spatial *Gestalt* is a Slepian principal quantity in facet B of scientific theory. This is because all appearances of objects are presented in human consciousness and space goes into the makeup of our deepest underlying concepts of all objects. Space is part of the makeup of human experience and representative *Gestalt* intimately determines what we can know of facet A. But it is quite wrong to regard this pure form of intuition as if it were an innate geometry – there is no innate geometry pre-wired into our brains – and even more wrong to regard space as some kind of set of mental cookie cutters that "stamp out" intuitions from a cookie dough of *materia sensibilibus*. A theoretical understanding of space in intuition requires a *Realerklärung* of the process by which spatial form is synthesized, i.e. the process of *Gestaltung* we call the pure intuition of space (or, if one prefers, the *pure intuiting* of space). We will deal with the exposition of this *Realerklärung* in the following subsection. But before doing so, we must examine the character of its end product since speculation of how any process works is fruitless if we do not first understand what that

process produces.

We begin with the two main conclusions of the transcendental Aesthetic of space:

(a) Space imposes no property at all of any things regarded as they are in themselves, nor any relationship of them to each other, i.e. no determination of them that attaches to objects themselves and that would remain even if one were to abstract from all subjective conditions of intuition. For neither absolute nor relative determinations can be intuited prior to the *Dasein* of the things to which they pertain, thus be intuited *a priori*.

(b) Space is nothing other than merely the form of all appearances of outer sense, i.e. the subjective condition of sensibility, under which alone outer intuition is possible for us. . .

We can accordingly speak of space, extended beings, and so on, only from the human standpoint. If we depart from the subjective condition under which alone we can come to outer intuition, namely that through which we can be affected by objects, then the representation of space signifies nothing at all. This predicate is attributed to things only insofar as they appear to us, i.e. are objects of sensibility. [KANT1: B42-43]

With these two *judicial* principles firmly in mind, we can proceed to a theoretical representation of the idea of space. Such a representation requires four considerations, one from each of the general ideas of Quantity, Quality, Relation, and Modality. We begin with Relation. Here the idea of space is an idea of transitive Relation:

Space is not an empirical concept taken from outer experiences. For in order for certain sensations to be related to something outside me (i.e. to something in another place in space from that in which I find myself), thus in order for me to represent them as outside and next to one another, thus not merely as different but as in different places, the representation of space must already be their ground. Thus the representation of space cannot be obtained from the relationships of outer appearance through experience, but this outer experience is itself first possible only through this representation. [KANT1: B38]

In regard to Relation, space is a capacity in sensibility common to all representations of objects of outer experience and common as well to all representations of outer relationships among these objects. It is "something that both belongs to and does not belong to" objects *as we know them*, something that pertains to the Nature of co-existing objects of outer appearance and to the determination of their reciprocal relationships. This is an idea of transitive Relation.

Next we consider the Modality of the representation of space:

Space is a necessary representation *a priori* that is the ground of all outer intuitions. One can never make a representation that there is no space, though one can very well think that there are no objects to be encountered in it. It is therefore to be seen as the condition of the possibility of appearances and not as a determination dependent on them, and is an *a priori* representation that necessarily grounds outer appearances. [KANT1: B38-39]

This is an idea of determining factor in Modality. It is worth our while to note something here that may be easy to miss. Kant tells us the representation of space is the ground of *all* outer intuitions. It is easy and even tempting to regard the explanations of spatial *Gestalt* in terms of one or only two of the classical senses, namely vision and touch. But to do so is an error. Space is

the form of *all* outer appearances for *all* sensory modalities. It applies as much to taste or smell as to seeing, hearing, or touching. This alone should suffice to warn us that subjective space is not, as many of Kant's readers have assumed, a geometrical space, a metric space, or a projective space. The *Gestalt* of space is something more primitive than any of these and necessary *a priori* for the possibility of any of these to be constructed in mathematics. It is also worthwhile to recall that the term "*a priori*" means only "prior to experience."

In our next description of the general ideas of subjective space, it may sound at first as if Kant is describing the Quantity of space but this is not so. Rather, for the *Quality* of space we have:

Space is no discursive or, as we say, general concept of relationships of things in general, but is a pure intuition. For, first, one can only represent a single space, and if one speaks of many spaces, one understands by that only parts of one and the same space. These parts cannot as it were precede the single all-encompassing space as its components (from which its composition would be possible), but rather are only thought *in* it. It is essentially single; the manifold in it, thus also the general idea of spaces in general, rests merely on restrictions. From this it follows that an *a priori* intuition (which is not empirical) grounds all concepts of it. [KANT1: B39]

Space is not *composed of* an aggregation of parts, i.e. not constructed by putting sub-spaces together to form it. Rather, sub-spaces can only be conceptualized and thought by mental operations that extract them *from* it by means of laying down particular restrictions. Since what we speak of here is explicitly not an idea of aggregation, this is enough to tell us the idea being expressed by Kant in the quote above is not an idea of Quantity. The possibility of *thinking* different sub-spaces *within* a single, all-encompassing space calls for notions of limitations and restrictions, and so what Kant has described in the quote above falls under the general idea of subcontrarity in representation, and subcontrarity is an idea of Quality.

Finally, for the idea of Quantity in the representation of space we have:

Space is pictured as an infinite *given* magnitude. Now one must, to be sure, think of every concept as a representation that is contained in an infinite aggregate of different possible representations (as their common mark), which thus contains these *under itself*, but no concept, as such, can be thought as if it contained an infinite aggregate of representations *within itself*. Nevertheless, space is so thought (for all the parts of space, even to infinity, are simultaneous). Therefore the original representation of space is an *a priori intuition*, not a *concept*. [KANT1: B39-40]

Note carefully that what we have here is a description of "how we see space" when we think about it. But this picture, this conceptualization, is a concept representation (thinking is cognition through concepts). The possibility of thinking about *objective* space in this way is possible only if there is a pure intuition (subjective space) up to the task. If there were not boundlessness in the progress of empirical intuition, no concepts of relationships could bring us to even a subjective principle of mathematical induction to infinity. This "un-end-able" (*unendlich*, "infinite") property of the form of appearances of outer sense is an integrative capacity of intuition; the

general idea of Quantity for space is integration.

Thus we have our 2LAR of subjective space as {integration, subcontrarity, transitive Relation, determining factor} in the representation of the form of empirical intuitions. Later in this book we will introduce twelve non-primitive logical *momenta* of judgment, which are the *momenta* of logical structuring in making combinations of concepts. These twelve *momenta* bear familiar sounding names and the traditional theory of classical logic can be re-cast in terms of these *momenta*.⁶ The *logical character* of space is then expressed in 2LAR form, from the theoretical Standpoint, by mapping the general ideas of representation above over to these logical *momenta*. When this is done, we obtain

$$\text{space} = \{\text{universal, infinite, disjunctive, apodictic}\}$$

as the logical character for the form of intuition of outer sense. We may note that this logical character of space accords well with how we typically think about space in physical terms.

This is the outcome the process of the pure intuition of space is capable of producing. Now we must look at this process itself.

§ 2.2 The Pure Intuition of Space

In a manner of speaking, the *process* of the pure intuition of space "stands behind" the outcome we have just discussed. Space, as form of outer appearances, is a principal quantity but the process that *makes* this representation of form does not appear to us in experience. Thus it lies outside the overlap with Slepian's facet A and therefore can be regarded only as a secondary quantity. How, then, shall we understand it and do so with sufficient grounds for the objective validity of this understanding? We must not leap immediately into mathematical speculation about the synthesis of the pure intuition of space, nor can the understanding we seek begin from the theoretical Standpoint. In his *Opus Postumum* Kant writes:

Without laws no experience can take place and, without a principle of the combination of the manifold in *a priori* intuition, no law. For Knowledge [*Wissen*] exceeds judgment and only makes this capable of thorough-going determination; receptivity discovers certainty in synthetic *a priori* judgments only if the objects of intuition first qualify for this, merely as appearance in my consciousness of myself. For this makes the formal which, pure of everything empirical merely in understanding, assembles rather than sets out a manifold of intuition inasmuch as it emerges from the subject's activity. Hence space is not a thing or object-matter [*Sache*], and places in it, as points, cannot be aggregated; they all coalesce into one point. [KANT: 22 (36-37)]

Acts of determining judgment produce objective judgments but only in the form of what we must call "local laws." Determining judgment finds particulars to be subsumed under general

⁶ This re-casting of traditional logic is done to make logic epistemology-centered rather than ontology-centered. Such a re-centering must be done in order to bring objective validity to formal logic.

concepts but is not itself capable of making the general concept. This task falls to the act of reflective judgment, which is tasked with *making a system of Nature*. Furthermore, no representation of sensibility becomes an intuition except that it be so-marked by reflective judgment at a moment in time. The Standpoint from which we must take our understanding of the pure intuition of space is the *judicial* Standpoint.

We must further note Kant's statement that "the formal" (that is, the synthesis of the form of an intuition) "assembles rather than sets out a manifold of intuition inasmuch as it emerges from the subject's activity." It is impetuous reflective judgment that forms the bridge between representation in sensibility and the Organized Being's actions in motoregulatory expression. Now, we have no perception of our motor acts in terms of signals going to the brain's motor centers (e.g. the motor cortices or motor centers in the brain stem). Early theories proposing this, called the feeling of innervation by nineteenth century psychologists, have long been proven false [JAME1b: 493ff]. If, then, it is to be possible for the Organized Being to "assemble rather than set out" the pure form of intuition "as it emerges from the subject's activity," there must exist some source of *materia circa quam* containing information about this activity. This is because information theory tells us that once information is lost it cannot be recovered again by any means whatsoever.⁷

The source of this information is easily identified from Figure 3.1.1. It is the kinaesthetic feedback of motoregulatory expression. We know of the *Dasein* of kinaesthetic feedback from studies of *soma* where it appears as, for example, the activities of sensory neurons with nerve endings in the intrafusal muscle fibers of the skeletal muscles. We know of the rational necessity of the *Dasein* of this source of information from Critical epistemology. Putting all this together leads us to our first theorem regarding the pure intuition of space: **spatial form is synthesized from the *materia circa quam* of kinaesthetic feedback in receptivity.**

There is another aspect of spatial synthesis, this one of a more purely mathematical nature, we can extract from Kant's statement above. This has to do with the idea of a mathematical "point." What is a "point"? The classical definition of a mathematical point goes back to the work of Euclid, who defined a "point" as "that which has no part." This original definition is more or less preserved by modern mathematics, which defines a "point" as "an element of geometry having position but no magnitude." The modern definition is the result of making an abstraction from an induction, the most common of which is the one where one starts with an object – let us say a ball – and imagines this object growing smaller and smaller in size without end. In the "limit" of this

⁷ Data can be lost without information being necessarily lost along with it. Data loss with information preservation requires that the data representations have the property of redundancy [WELL2: 49-51].

induction, the "size" of the ball is regarded as becoming infinitesimal. If we examine how we look at this induction, it is not difficult to realize that we *think about* the "limit" as being reached when, so to speak, the ball "becomes so small that if it became any smaller it would cease to exist at all."

Now, if you feel a little uncomfortable about this mathematical picture of a "point," there is some comfort to be had in knowing that from time to time mathematicians do too. Like other scientists in their fields, most mathematicians spend little or no time "philosophizing" about mathematics. But from time to time some of them do and, often enough, what tends to come out of this (when anything at all does) are interesting new ways to look at mathematics. In the case of the "point," one such interesting alternative view is the "non-standard analysis" of Robinson.⁸ There is much in Robinson's model that at least hints at a congruence between non-standard analysis and set membership theory, although at the time this is being written there has not been any published attempt to establish a connection between the two approaches.

Under Critical epistemology, the classical mathematical idea of a "point" lacks objective validity. It is one thing for mathematicians to use the idea of a point in doing pure mathematics; it is something else altogether when it comes to applying such pure mathematics to Nature. The issues inherent in this come down to the axioms mathematicians use. In the *CPPM* (Chapters 23 and 24), it was pointed out that in order to have objective validity (that is, to be applicable to Nature in science) the axioms of mathematics must first be grounded in the *acroams* of Critical metaphysics. In *CPPM* the author called for the development of mathematics to be re-organized into two levels, namely Critical mathematics (which would deal with principal quantities and be applicable to Nature) and hypothetical mathematics (which could be used for, and only for, secondary quantities). Critical mathematics would be mathematics that used only axioms deducible from or with objective validity under the Critical *acroams*. At the time of this writing, this re-organization of mathematics has not yet begun and remains a key task for the future.

The purpose in bringing all this up here is simply this: We must not regard Kant's remark about "places in space coalescing into one point" as literally referring to the classic mathematical point. When we understand that the somatic correlate to the representation of space (the pure form of outer appearances) rests with signals functioning as the kinaesthetic feedback of motor-regulatory expression, the issues that arise in asking how the ensemble of such signals could come to be regarded as any sort of "points" in the *mathematical* expression of the representation of space become obvious rather quickly. Critically, a "point" is a singular representation regarded

⁸ Abraham Robinson, *Non-standard Analysis*, revised edition, Princeton, NJ: Princeton University Press, 1996.

as having *intensive magnitude*, a technical term we will deal with in more detail later.⁹

An applied metaphysic – even a Critical applied metaphysic – can take us only so far. Metaphysics cannot replace science nor does it try to do so. It merely serves to establish ground rules for objective validity in scientific (and mathematical) theories. An applied metaphysic is a bridge between Critical metaphysics proper and the particular topic of one of the special sciences. In regard to the synthesis of the pure intuition of space, the Critical analysis, made from the judicial Standpoint in *CPPM*, leads us to understand the pure intuition (pure intuiting) of space as the following: **the pure intuition of space is a topological synthesis**. This is to say *the process* of the pure intuiting of space is a process that, when coupled with the process of reflective judgment, *produces a synthesis in which the data of the senses is assimilated into a topological structure*.

Epistemological conclusions such as this have implications testable in the crucible of scientific experience, and when we look for this evidence we in fact find it. This was reviewed in Chapter 17 of *CPPM* and we will not re-cover that lengthy discussion here. What we will and must make clear here is the following. The topological synthesis of space is *not* a ready-made topology-in-pre-formed-*Existenz* within the Organized Being on its natal day. Such an *a priori* innate topology is neither necessary for the possibility of experience nor is it supported by any currently known empirical evidence. Rather, mental physics requires only the capacity to synthesize, in sensible representation, representations that *make possible the generation of a system* of sensible representation that can *come to satisfy* the definition of a topological structure. The synthesis of the pure intuition of space is a *structuring*, not the structure itself. Only the *ability* is necessary *a priori*. The structure itself "emerges from the subject's activities." A corollary to this is that we should expect there to be diversity among individual human beings, existing alongside equally expected commonalities, in terms of how each one of us "pictures space" in intuition.

§ 2.3 The Topological Synthesis of Space

The *Penguin Dictionary of Mathematics* defines topology as "the study of those properties of geometric figures that are invariant under continuous deformation." Topology is also often called "rubber sheet geometry." However, although the kinship between topology and geometry is a close one, topology and geometry are not one and the same thing. In some ways – many ways, in fact – topology can be regarded as a kind of pre-geometry in the sense that it is entirely unconcerned with topics such as distances, angles, congruence or similarity of triangles, projective relationships, and the like.

⁹ The impatient reader may consult the technical Glossary at the end of this book and Chapter 17 of *CPPM*.

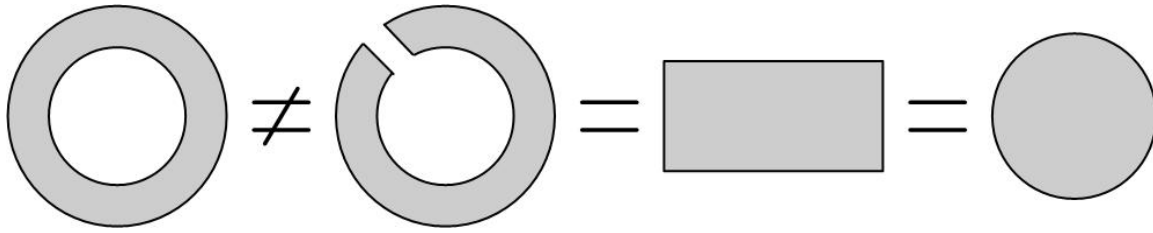


Figure 3.2.1: Topology examples. The three right-most figures are topologically identical and unequal to the left-most figure.

Topology deals instead with such ideas as proximity, continuity, connectedness and separation, boundedness, and enclosure. Figure 3.2.1 illustrates four geometrical figures. To a topologist, the left-most figure – a donut – is topologically inequivalent to the three right-most figures, all of which are topologically equivalent. The left-most figure divides the plane of the paper into three regions: outside the donut, inside the donut, and the annulus of the donut itself. The three right-most figures, on the other hand, divide the plane of the paper into only two regions. This and some other properties shared by the three figures are what make them topologically equivalent. Extensive psychological studies have brought forth the important finding that the earliest perceptual relationships grasped by infants and young children are topological rather than geometrical relationships, and that the child's construction of the idea of objective space proceeds from and builds upon more primitive topological relationships [PIAG3].

Again, though, we remind ourselves that it is not some one innate and privileged topological system that constitutes the pure intuition of space but, rather, the pure intuition of space is the ability to *construct* a topological system. This is, once more, "know-how" knowledge *a priori* and not objective knowledge *a priori*. The latter, objective knowledge *a priori*, is what was called "innate knowledge" by the philosophers of the rationalist school; both Critical epistemology and empirical psychology reject the transcendental *Dasein* of this sort of innate knowledge.

At the time of this writing, the topological theory of the pure intuition of space is undeveloped and it can be anticipated that much labor will be necessary in order to bring it forth. Here in this book about the *principles* of mental physics, our task and objective is to lay the groundwork for this future development. In doing so, the first thing we must keep in mind is that the mathematical objects of this mathematical theory are, one and all, Slepian secondary quantities. Nonetheless – and this is the second thing we must bear in mind – these secondary quantities must eventually lead to and match up with Slepian principal quantities, and so we are not free to adopt the attitude of the formalists and ignore the epistemological constraints that must apply even to these secondary quantities.

In formal mathematics a **topological space** is defined to be a set X of points x with a collection B of subsets $N \subseteq X$, called *neighborhoods*, such that the following properties hold:

1. every point x is in some neighborhood, i.e.,

$$\forall x \in X, \exists N \in B \text{ such that } x \in N;$$
¹⁰
2. the intersection of any two neighborhoods of a point contains a neighborhood of the point, i.e.,

$$\forall N_1, N_2 \in B \text{ with } x \in N_1 \cap N_2, \exists N_3 \in B \text{ such that } x \in N_3 \subseteq N_1 \cap N_2.$$

The set B of the neighborhoods N is called the basis for the topology on X .

This mathematical definition defines a topological space. A *topology*, T , is the set of all "open sets" $O \subseteq X$. A set O is an open set if for each $x \in O$ there is a neighborhood $N \in B$ such that $x \in N$ and $N \subseteq O$. For the non-mathematician, what is important about this definition is that it means that any particular topological space can be home to a manifold of different topologies. A topology is one particular representation of a topological space.

As non-obvious as it will seem to the non-mathematician, the definitions presented above lead to an important and very basic theorem in topology. Let us use the symbol $\emptyset$ to denote what mathematicians call "the null set" or, equivalently, "the empty set" (the set with no members in it). Let X be a topological space with topology T and basis B . Then all of the following statements are true:

1. X and $\emptyset$ are members of T ;
2. the union of any collection of members O in T is in T ;
3. the intersection of any finite collection of members O in T is in T .

A corollary to this theorem is that a set O is an open set (and therefore a member of T) if and only if O can be written as a union of members N of the basis B .

It would not be surprising if, to those of us who are not professional mathematicians, these definitions and theorems, heavily couched in the language of mathematics, seem as opaque as a granite stone. We will need some of math's symbolic shorthand for what follows, but we will not be wandering off into the jungle of mathematics proper in this book. This is not because your author is not tempted to do so; it is because at the time of this writing next to nothing has been done to blaze trails in this jungle for us to follow. We have instead a very important task to accomplish here, namely to set down the Critical ground rules to which such a future trailblazing must adhere from the very start.

¹⁰ These perhaps unfamiliar symbols are part of the mathematical shorthand first introduced by Whitehead and Russell in their unreadable masterpiece, *Principia Mathematica*. The English translation of these mathematical "sentences" is given in the text above them. The reason mathematicians employ these symbols is because mathematics is a language for saying things very precisely and the symbols help mathematicians to do this.

The Critical deduction of these ground rules calls upon the judicial Standpoint in our Critical applied metaphysic and this deduction is not brief. The details of it can be found in Chapter 17 of *CPPM* and we will do no more than state and summarize them in this book.

§ 2.3.1 The Universe X and the Points x

The mathematical formalism above belongs to the branch of topology theory called "point set topology." In the technical language of mathematics, the overall set X is usually called "the universe" (of the mathematical discourse) and the mathematical objects x are called "points" (which more or less illustrates the eventual connection between topology and geometry that a mathematician often wishes to forge). A mathematician will not usually concern herself with "what x is" or "what X is" (beyond the understanding that X is some aggregate of "points" x). She regards these quantities as primitives and uses some system of axioms to fix what some formalists have called "the rules of the game." Given the axioms and some carefully defined and delimited mathematical objects, the mathematician is occupied only with finding out what is true about these objects under the "rules" provided by the axioms. Mathematical truth is what the professional mathematician seeks (these truths are called "theorems"), and often the mathematician neither knows nor cares what applications – if any – there might be for these mathematical truths outside of the field of pure mathematics. Mathematicians – at least most of them – do mathematics for the sake of mathematics. It just so happens that what they do turns out to be of immense value to the rest of us when we can properly figure out how to use the tools they provide.

Mathematical objects, systems, equations, and theorems do not come with an owner's manual for the rest of us to use. No equation or theorem comes with a tag that says, "Use me here and here, but not there or there." If I am a scientist and I wish to employ the power of mathematics in my science, it is up to me to figure out what mathematical objects, systems and theorems are objectively applicable to the scientific questions I am interested in answering. This is how it should be when one considers that there are more than enough purely mathematical problems yet to be solved to occupy every living mathematician for a lifetime. Our first important question here is: How, if at all, can we apply the mathematician's X and x to the problem of the synthesis of the pure intuition of space? Put another way, what do X and x *represent* in the context of the pure intuition of space? We know they are secondary quantities; we know they have carefully designed and crafted mathematical properties; but what do they *mean* to us?

A good place to begin is to determine what they are not. Neither x nor X represent a sensation or a feeling. Sensations and feelings are the *materia in qua* of intuitions and affective perceptions.

From the theoretical Standpoint, sensation can be regarded as nothing else than the effect of an object on the Organized Being's capacity for representing how it is affected by that object. Furthermore, sensation and feeling imply consciousness and we are not conscious of the *materia circa quam* of intuition.

An objectively valid *Realerklärung* of x and X must call upon the judicial Standpoint and the applied metaphysic of the data of the senses in *psyche* (chapter 6 of *CPPM*; chapter 4 in this book). The general ideas of representation for space are

space = {integration, subcontrarity, transitive Relation, determining factor}

and the applied metaphysic for the data of the senses when applied to space aligns the Quantity, Quality, and Relation of space with the necessary *materia circa quam* of its representation. (The general idea of Modality is different because sensibility does not judge and is therefore not a determining factor; the synthesis of the pure intuition of space deals only with the determinable in perception).

In Critical epistemology, an **occurrence**¹¹ is a single act with its result (a totality, thus an idea falling under the idea of integration in Quantity). A **circumstance** is the outer connection in which the occurrence happens (the "universe" of occurrences). The representations of x meet this first definition (act of receptivity resulting in a possible representation of outer sense) and so we can call x a *sensuous occurrence* and X the *sensuous circumstance*. When we regard the Organized Being as an Object in Nature and in the context of the pure intuition of space, its capacity of receptivity, in terms of Quantity, is a whole of possibilities for the manner in which its outer sense is co-determined in reciprocity with its environment. This is an idea of integration that here bespeaks of composition of the form of what we may call "accidents of the senses." In Quantity, a specific x is a specific sensuous occurrence and X is the "universe" of all such possible occurrences.

The Organized Being cannot be said to have "received" anything in receptivity unless this "reception" can be viewed as a ground for consciousness of a change in its state of being. Consciousness is the Organized Being's representation that another representation is present (the presentation of a representation), and this representation requires an act of judgment (sensibility does not judge). The Quality of x is therefore to be regarded as an inducement for a reflective judgment (which is an idea of subcontrarity: the matter of occurrence as the possible cause¹² of a

¹¹ The etymology of the Critical term "occurrence" traces to the Latin word *eventus* (an event, occurrence).

¹² The notion of *cause* is a notion of Quality (matter of the matter of representation) because a cause is regarded as matter, i.e. ground for an effect. *Causality* is a notion of Relation, specifically the notion of the

change in circumstance that reflective judgment might or might not effect; some occurrences are causes and some occurrences are-not causes). The common coalition (matter of composition) represented by "points" x in X is the registering of a possible conscious effect of receptivity. Sensibility is judged by reflective judgment, which deals solely with affective perception. It follows that the Quality of x is coalition of representation with a sensuous degree of *Lust per se*.¹³

The transitive Relation for the data of the senses is called the **principle of emergent properties**. Let us recall that *nexus* deals with the connection of heterogeneous factors regarded as nonetheless necessarily belonging to each other. In our present context of the pure intuition of space (which belongs to sensibility), the adjoined factor is reflective judgment (which does not belong to sensibility). The necessary connection between them is the phenomenon of *consciousness*, and seen in this context x is easily seen to be that which can possibly emerge from obscure (unconscious) representation in a perception. The gathering of a manifold of representations x to form a disjunctive part, N , of an overall manifold of possible perception, X , therefore constitutes in the form of connection what we can call **the focus of Attention**. Put into different words, the possibility of the form of X subsists in the formation (*Gestaltung*) of subsets N where such subsets are regarded as constituting matters of the act of Attention.¹⁴

Modality in the representation of X (X being regarded as sensuous circumstance = the manifold of occurrences x) falls under the idea of the determinable in the data of the senses. We do not say the sensuous occurrences x in and of themselves determine the acts and actions of the Organized Being. Rather, it is the full interplay of receptivity, judgmentation, practical choice, motoregulatory expression, and the *Lust-Kraft* of adaptive *psyche* that combine to *produce the circumstance*. X is that in sensibility which is determinable through the full interplay of the noetic, psychic, and somatic capacities of the Organized Being. "What" is determinable through the synthesis of apprehension is the state of satisfaction or dissatisfaction in the data of the senses (CPPM, Chapter 6) and so X is the determinable form of sense in general.

The ideas of x and X stand as matter (x) and form (X) to each other, and neither has meaning or context without the other. Jointly they form the 2LAR shown in Figure 3.2.2 below. We can see from our discussion here how very little x resembles a classic geometric point. The two have in common only the fact that each is regarded as a singular quantity. Space is not a geometry.

determination of a change by which the change is established according to general rules. The *rule* for determination of the change under the condition of a cause is called the **causatum**.

¹³ Recall that *Lust per se* is the fundamental capacity of *psyche* for determining an adaptation.

¹⁴ Critical epistemology draws a technical distinction between the terms Attention and attention. The first (in Kant's terminology, *Attention*) is the act whereby a representation is made clear and conscious while other representations remain obscure. The second, attention (*Aufmerksamkeit*), means consciousness according to choice.

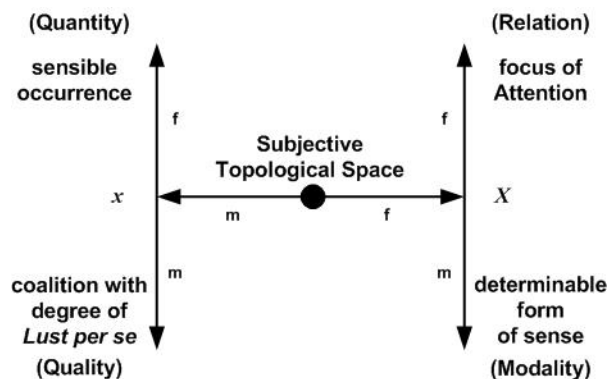


Figure 3.2.2: 2LAR of subjective topological space.

§ 2.3.2 The Topological Synthesis

The practical *Realerklärung* of the pure intuition of space is that it is the faculty of rules for the construction of a topology, T . The pure *a priori* intuition of space is not some pre-defined topology but, rather, the capacity to *build* a systematic topology. What does this mean?

In Kant's day topology theory did not yet exist in mathematics and so Kant himself provides us with little help in understanding this. He did, however, provide us with an example of an application in his *Metaphysical Foundations of Natural Science* [KANT4]. The example is Kant's exposition of phoronomy (which, in impoverished form, physics today calls "kinematics"). *CPPM* (Chapter 17) made use of this example in deducing what follows here.

Let $x^{(1)}$ denote a sensuous occurrence. The occurrence implicates a possible reflective judgment leading to a motor act a_1 (through the motoregulatory expression of *psyche*) as well as an act of imagination, s_1 , in the synthesis of apprehension. Motoregulatory expression in turn implicates an effect, r_1 , on receptivity. Symbolically, we can write this as $x^{(1)} \rightarrow \langle a_1, s_1, r_1 \rangle$. We can call this formula a *specific circumstance of $x^{(1)}$* . Because the specific circumstance involves motoregulatory action and receptivity, it implicates another occurrence, which we can write as $\langle a_1, s_1, r_1 \rangle \rightarrow x^{(2)}$. Clearly this process can be continued indefinitely.

Now let us look at the forms, which we may call *effective accidents in sensibility*, that are possible in this. Two obvious ones are the singletons $O_1 = \{x^{(1)}\}$ and $O_2 = \{x^{(2)}\}$. Another is that in the matter of the occurrences that may be common to both, i.e., $O_3 = O_1 \cap O_2$. Here we should note that it is possible the two occurrences might share nothing in common, in which case the intersect operation would, mathematically, produce a null result. We interpret this by saying the effective accident O_3 is *impossible*, i.e., not givable in sensibility.

Fourth and fifth possibilities are provided by differences $O_4 = O_1 - O_2$ and $O_5 = O_2 - O_1$. Here we are using a mathematical notation convention called "the set difference." It is defined thusly. If $Y = \{a, b, c\}$ and $Z = \{a, b, d, e\}$ then $Y - Z = \{c\}$ and $Z - Y = \{d, e\}$. Note that set difference is

generally not commutative. It is also possible for a set difference to be null and in this case we again have an impossible effective accident. It is noteworthy that in the earliest stages of life the sensorimotor capacities of a human infant are uncoordinated (e.g. seeing is uncoordinated with prehension). This lack of coordination, clearly observable in behaviors, is indicative of impossible effective accidents in sensibility. That such coordination later becomes possible is attributed to the entry into the synthesis of apprehension of concepts introduced through the synthesis of reproductive imagination following the construction of the manifold of concepts connected by acts of determining judgment. We may further note that the set difference operation produces subsets, i.e. $O_4 \subseteq O_1$ and $O_5 \subseteq O_2$. Finally, we may have the union $O_6 = O_1 \cup O_2$ as a possible effective accident.

It is easy to see that these mathematical operations make possible the conditions for a topology presented earlier, although they provide only necessary and not sufficient conditions for the synthesis of the pure intuition of space to yield a topology. Completion of the theory requires a theory we could call **topology generation theory**. This would be a new exercise in and development for mathematics inasmuch as mathematics typically confines itself to topological *analysis* from a starting point of some assumed universe X and basis B , and generally confines this work to applications to *closed* systems (X, T) . In the topological synthesis of the pure intuition of space, (X, T) is an *open* system, the *Gestaltung* of which grows by experience.

So far as I know, a general mathematical theory of topology generation does not presently exist, although I am not a mathematician and I would not be shocked to learn that one is actually hiding somewhere in what is, for me, the impenetrable jungle of mathematics literature. It seems clear that a topology generation theory is at least possible since mathematicians do manage to come up with topologies to study and generalize. If they can do it in the particular, they should be able to figure out more generally how they accomplish the particular and teach this to us.

However, mathematicians work from a set of conventionally established axioms, and it was shown in *CPPM* that the principal axiom systems (e.g. the Zermelo-Fraenkel-Skolem axioms of set theory) contain axioms that lack objective validity. Standard topology theory is based on such an axiom system and, therefore, while it is a system of what I call hypothetical mathematics, it is not a system of Critical mathematics. Whether hiding within it is some piece employing only axioms that stand as objectively valid under the Critical acroams remains to be seen. Even though the topological synthesis involves only secondary quantities, it must nonetheless be productive of outcomes capable of standing as Slepian principal quantities. Principal quantities *are* constrained by the acroams of Critical metaphysics and so a theory of topology generation as envisioned here does not presently exist inasmuch as the objectively valid transformation from secondary

quantities to principal quantities has not been worked out. It can be hoped that before too many more years have passed some cadre of capable mathematicians will step forward to take up this challenge. Einstein said, "So far as the theories of mathematics are about reality, they are not certain; so far as they are certain, they are not about reality." This is not necessarily true. Critical mathematics – when we finally have it in hand – will both be about reality and will be certain because it will be grounded in the very acroams that specify what it is to be real and what it is to be certain. It is only in hypotheticalal mathematics that Einstein's dictum is true.

§ 2.3.3 *Soma* and Space

It is a theorem of the Critical metaphysics of the phenomenon of mind that to every mental representation there corresponds a somatic one. The mind-body division is merely a logical division. It is therefore appropriate to close this section with a few words on this topic.

All that we have experienced in regard to *soma* and all that we shall ever experience in regard to *soma* is placed squarely in Slepian's facet A of human knowledge. The discerning reader may well be wondering by now, "How can the vast *complexus* of electrical, chemical, biophysical, and anatomical phenomena found in *soma* be even remotely set beside the constructs presented above?" The response to this question lies with the observation that, insofar as our understanding of somatic signaling is clothed in mathematics, the mathematical quantities that bring us our immediate understanding of *soma* must, if these ideas are to be objectively valid, consist entirely and only of principal quantities. We are thus brought immediately back to the urgent need to work out the system of Critical mathematics.

The quantities x , X , B , N , and O employed above are secondary quantities. Our pathway to link them to principal quantities – and from there to actual experience with *soma* – takes its point of departure from the 2LAR of topological space. When we have finally succeeded in recognizing the **somatic appearance** of subjective space, it will be *the phenomenon of spatial form in perception* that we recognize and our understanding of the phenomenon will be a *practical* understanding *viewed from the judicial Standpoint* that we achieve. As an Object, subjective space is a mental Object – which means subjective space *per se* is supersensible – and it will always remain so. Appearances in *soma* can give us only what we might call the "symptoms" of the *Gestalt* of the pure form of outer sense. We know the real *Dasein* of space only from its transcendental position of necessity for the possibility of human experience as humans come to have and know experience. All we shall ever know of its details and particulars will be knowledge of its empirical *Existenz* (and this is practical knowledge). The intelligible nature of the pure intuition of space guarantees that our understanding of its *Existenz* will be mathematical.

The principal quantities of this mathematical understanding will, of course, have testable implications since by definition principal quantities must overlap facet A of experience. The secondary quantities remain remote, just as do the secondary quantities physicists employ in the theory of quantum electrodynamics remain remote from such phenomena as are credited to mass and electric charge.

Some neural network theorists, most notably Malsburg¹⁵, have conjectured that perhaps the neural network system of the brain has a topological structure. Nothing much has yet come of this conjecture, and one reason for this can likely be pinned on the fact that this conjecture is based on rather vague notions of optimal connectivity that lack clear practical ideas of what sort of "optimality" we are talking about. For example, Malsburg would assign the role of "point" to the individual neuron, that of "neighborhood" to connectivity among neurons, and the role of neuronal signaling to that of a "correlate" of a topological neural network. Without disrespect to this noted theorist, this picture is a mere product of analogy and lacks a known basis for its objective validity. However, the *function* of topological structuring must certainly exist, however latently, in neuronal organization since the pure intuition of space is a transcendental necessity. Objectively valid research into this topic must begin with Slepian's principle, and here the likelihood seems to be that topological "points in space" will find their description from set membership structures or, perhaps, Robinson's non-standard analysis rather than from the "crisp" analogy with geometry and its objectively invalid concept of a "point" as the limiting case of a beach ball shrinking to the infinitesimal.

§ 3. The Transcendental Aesthetic of Time

The metaphysical difficulties and controversies attending the question "What is space?" are matched and perhaps even exceeded by difficulties and controversies swirling around the question "What is time?" *The Oxford Dictionary of Philosophy* tells us, "The nature of time has been one of the major problems of philosophy since antiquity," and then goes on to devote a half page to admiring the problem without providing any answer or taking any position on it. Reber's *Dictionary of Psychology* calls time "a slippery concept indeed" and goes on to reference the modern-day Einsteinian idea of "space-time":

In this conception, time is not a separate phenomenon and it becomes metaphysically meaningless to talk of the 'flow' of time or the 'passage' of time. Rather, the full spatio-temporal structure of things is given by the changing distribution of matter.

¹⁵ Christoph von der Malsburg, "The correlation theory of brain function," 4th revision, in *Models of Neural Networks II*, E. Domany, J.L. van Hemmen, and K. Schulten (Eds.), NY: Springer-Verlag, 1994, pp. 95-119.

The reason for going into this problem here is that this relational perspective is precisely the one psychologists adopt, perhaps without knowing it, when they speak of time, and it is the only even marginally satisfactory way in which to conceptualize the psychological experiencing of time, duration, extension and the varied connotations of words like *past*, *present*, *future*, *now* and *then*. Put simply, from a psychological point of view time is always dealt with relativistically: time is marked and phenomenologically experienced as events occurring relative to other events.

Despite a hint of circular argument latent in this explanation, we will see that Reber isn't too far off here in his treatment of time. However, we will find this is not "the only even marginally satisfactory way" to understand time.

The Oxford Dictionary of Physics defines time as "A dimension that enables two otherwise identical events that occur at the same point in space to be distinguished. The interval between two such events forms the basis of time measurement." It then goes on to present a kind of revisionist history lesson about time, the main point of which seeming to be that Einstein didn't really mean it when he flatly said all time is relative and the only meaning the concept can have for physics is one couched in the procedure by which time is measured by clocks. This revisionist history lesson's main point seems to be an attempt to argue that Newton wasn't as wrong as Einstein said he was. In effect, it is a feeble attempt to restore an ontology-centered status to time as a thing rather than let time remain a practical idea. *The New Penguin Dictionary of Science* provides the accepted definition used in modern day physics:

One of the four coordinates of spacetime that define events. An event occurs at a particular position at a particular time. Time appears different from space, at least in the human perception of it, in that the world moves steadily forward in time.

Still, there are a couple of weaknesses in this definition. One is that human beings do not "perceive" time; time is not a direct object of the senses. Reber put it correctly; we "conceptualize the psychological experiencing" of time. The second weakness here is the statement that "the world moves steadily forward in time." Why is this a weakness? It is because the theory of quantum electrodynamics views, for example, the antiparticle called the positron as an electron *moving backwards in time*. In physics' view, not everything "in the world" moves steadily forward in time. Recently, a still-small number of physicists have even been engaging in Platonic romantic fantasies that "time travel might really be possible"; owing to a complete lack of any basis in any currently-possible empirical experience, I regard this kind of talk as pure Platonic hogwash. There seems to be a lot of that going on these days.

Owing to the generally abstract nature of mathematics, one might expect mathematicians to take no position on any ontological or epistemological considerations of "What is time?" But here they surprise us. *The Penguin Dictionary of Mathematics* defines time as "The continuous, irreversible passage of existence, or part of this continuum." We will see that if we take

"existence" to mean *Existenz* (rather than *Dasein*), this *explanation* is not too far off from the Critical solution to the question.

The Critical theory of time is begun in Chapter 17 and completed in Chapter 21 of *CPPM*. As was the case for space, we begin by drawing the Critical distinction between **objective time** and **subjective time**. Objective time is the "time" physicists deal with. It is a *noumenon* and its only objectively valid definition is a practical definition – the practical explanation, in fact, set out by Einstein in the relativity theory. Objective time is a mathematical object, a principal quantity, and its overlap with facet A of Slepian's model is provided by its *practical* operational definition in terms of clocks. All this is just the way non-speculative physics deals with and understands "time." To go beyond this practical definition and chase the inquiry "What is the *thing* we say our clocks are measuring?" is to shift to an ontology-centered pseudo-metaphysic and lose oneself in the fog and mists of transcendent illusion.

Subjective time – which we will hereafter just call "time" – is the pure form of intuition of inner sense. While space is the pure form of appearances in intuition, time is the form for both intuition and affective perceptions. Cognitions, which always are presented by intuitions, thus have both a spatial and a temporal constituency in their represented form. In this sense, intuitions can properly be said "to be in space-time" if "be in" is only taken to mean their form of representation. Exactly as was the case for space, and for exactly the same reason, the correct epistemological Standpoint for the theory of time is the judicial Standpoint. The *Realerklärung* of time can only be a practical explanation in terms of the manner in which we understand appearances in experience (loosely speaking, an explanation in terms of "what 'time' does").

In this *Realerklärung*, like that of space, the pure intuition of time is a secondary quantity. The principal quantity it underlies is *the perception of change* in experience. The experiences we may call "temporal experiences" appear to us in three *modi* we will call **the modi of time**. These are: (1) persistence in time; (2) succession in time; and (3) coexistence¹⁶ in time. What we will see is that the pure form of time is a mathematical **order structure** and the pure intuition of time is the **synthesis of an ordering structure** in experience. Time is *a priori* necessary for the possibility of experience as human beings come to know experience.

§ 3.1 The 2LAR of Time

The objective validity of the idea of time, like that of space, can only be practical objective validity and the judicial Standpoint is its proper Standpoint. Our *understanding* of time is, of

¹⁶ *Zugleichsein*, literally "to be at once." It means the *Existenz* of two or more objects contained in the same intuition and, therefore, "at the same moment in time."

course, a theoretical understanding (theoretical Standpoint) that must be deduced on the basis of practical considerations viewed from the judicial Standpoint. From the theoretical Standpoint, time is not a "substantial thing" (as, e.g., Newton tried to make it) but, rather, a mental capability. We represent this capability in 2LAR form as follows.

For Relation, Kant writes,

Time is not an empirical concept that is somehow deduced from an experience. For coexistence or succession would not themselves come into perception if the representation of time did not ground them *a priori*. Only under its presupposition can one represent: that several things be at one and the same time (conjointly) or in different times (one after another). [KANT1: B46]

Concepts, once represented, cannot properly be arranged temporally in the re-presentation by the synthesis of imagination unless it is possible for this temporal ordering to be pre-fixed in intuition. We cannot be "given" this ordering through the senses because time is not an object of outer sense. The three experiential *modi* of persistence in time, succession in time, and coexistence in time are characteristics of Relation *givable (dabile)* in sensibility only *a priori* through the pure intuition of inner sense. Time as Relation is Relation in inner sense, which is to say it is the internal Relation in representations. In the classical language of formal logic, time is *categorical* Relation.¹⁷

For Modality,

Time is a necessary representation that grounds all intuitions. In regard to appearances in general one cannot remove time, though one can very well take the appearance away from time. Time is therefore given *a priori*. In it alone is all actuality of appearances possible. The latter could be altogether omitted, but time itself (the universal condition of their possibility) cannot be removed. [KANT1: B46]

All appearances are said to "fill time" and, in this sense, it is tempting to regard time as some kind of container. But this is not a good simile and it is better to say time is the capacity to order presentations in sensibility in the manner human beings perceive Nature. Time as the matter of the form of the manifold of appearances provides the capacity to organize Nature with regard to temporal appearances (i.e., "to put appearances in their temporal places"). Time as Modality is determining factor.

For Quantity,

Time is no discursive or, as one calls it, general concept, but a pure form of sensuous intuition. Different times are only parts of one and the same time. Their representation, however, which can only be given through a single object, is an intuition. Furthermore,

¹⁷ We should note that *objective* time, operationally defined in terms of measurements using clocks, has the *external* Relation for its 2LAR when objective time is employed in the equations of science, and the transitive Relation for the operation of its empirical determination using clocks. The latter is called *duration*.

the proposition that different times cannot be conjoint cannot be derived from a general concept. The proposition is synthetic and cannot arise from concepts alone. It is therefore immediately contained in the intuition and representation of time. [KANT1: B47]

This characteristic of appearances in experience speaks to the integrative capacity of time, and thus the idea of Quantity for the 2LAR of time is integration. Time as Quantity is a totality and its logical character is that of the logically universal.

Finally, for the general idea of the Quality of time,

The infinitude of time means nothing more than that every determinate magnitude of time is only possible through limitations of a single time grounding it. The original representation *time* must therefore be given as unlimited. But where two parts themselves and every magnitude of an object can be determinately represented only through limitations, there the entire representation cannot be given through concepts (for they contain only partial representations), but immediate intuition must ground them. [KANT1: B47-48]

A moment *in* time must presuppose an "All-of-time" as its substratum. The presentational capacity of time works through particular limitations just as the presentational capacity of space works through limitations in an "All-of-space." The idea of the Quality of time is subcontrarity and time as Quality is judged as the logically infinite (*unendlich*, "un-end-able").

It is interesting to compare the logical *momenta* of space and of time. We have the 2LARs

space = {universal, infinite, disjunctive, necessary}
time = {universal, infinite, categorical, necessary}.

These differ only in Relation. It is the interplay of this difference that makes possible presentations of anticipations in reasoning. This is because the hypothetical (in succession) is the synthesis of the categorical seen as the disjunctive. The *Gestaltung* of intuition requires both the pure intuitions of space and time working together to make experience possible. It is in this sense that we may say the form of empirical intuitions is presented as space-time in a manner congruent with Reber's description of "time" quoted earlier. Because the pure intuitions of space and time are necessary for the possibility of representing any intuition or concept, including the *ideas* of objective space and time, subjective space and subjective time are the epistemological grounds of objective space and time. Seen in this light, Einstein's theory acquires an objectively valid grounding in Critical metaphysics and his pronouncement that physics must employ space-time, rather than space and time separately, in formulating all laws of physics gains apodictic confirmation in the Critical philosophy. Affective perceptions, on the other hand, have only time as the pure form of their representation in sensibility. This, we will see, has crucial consequences.

To sum up the logical character of the pure intuition of time, we have:

1. In **Quantity** – time is a universal form of sensuous intuition characterized by its integrative capacity;

2. In **Quality** – every determinate magnitude of time is only possible through limitations of a single time grounding these determinations, and this limitation is the presentational capacity of time; thus, its logical Quality is the infinite *momentum*;
3. In **Relation** – the *modi* of persistence, succession, and coexistence are givable only through a categorical and pure intuition of inner sense;
4. In **Modality** – time is a necessary representation that grounds all intuitions through a capacity to order presentations in sensibility.

Isaac Newton was correct to characterize "pure time" as *mathematical* and *absolute*; he erred in his ontology-centered placement of his "absolute time" as a thing external to and independent of the Organized Being who represents time. The pure intuition (intuiting) of time is a secondary quantity of facet B. In terms of Critical metaphysics proper, our *understanding* of the pure intuition of time calls upon the principles of Rational Physics and Rational Psychology.

The mere fact that the pure intuition of time is a mathematical secondary quantity does not leave us free to invent whatever sort of order structure we choose in our *Realerklärung* of the pure intuition of time. Rather, the explanation must conform to the acroams of Rational Physics and Rational Psychology as these acroams are viewed in the judicial Standpoint. Time carries the Modality of the determining factor in the pure form of inner sense, and we must understand the acroamatic requirements placed upon the Modality of time. From the logical-judicial Standpoint of Rational Physics, these acroams proclaim:

1. The representation in sensibility and the motor faculties of the Organized Being are such that the former can be joined to specific capacities for actions in the latter;
2. That which coheres with the material conditions of meanings (somatic motoregulatory expression) is actual;
3. Necessity takes its *Realerklärung* from regulation by practical Reason, which enforces coherence in meaning.

In addition to these, the transcendental-judicial Idea of Rational Psychology is

4. Unconditioned unity in apperception of all perceptions is the interrelationships of meaning.

Neither the pure intuition of time nor that of space makes the connection between sensibility and motoregulatory expression. That falls to the impetuous process of reflective judgment. The determination that a representation of sensibility is judged to have the necessary coherence with the acroamatic conditions under the principle of formal expedience is what the act of reflective judgment does in marking sensibility at a moment in time. These markings are the causeway to Slepian principal quantities in our mathematical theory of sensibility, and this theory must take that into its accounting. As will be seen in what follows, this requirement leads to a prominent role for intuitions in our theory.

§ 3.2 Partial Orderings and Order Structures

The pure intuition of time as a process is the synthesizing of order structures in sensibility. We therefore begin by reviewing what the mathematician means by an "ordering." An ordering consists of a set S with members s_1, s_2 , etc. and a binary relation R operating on this set. The term "binary" used here means that R is a mathematical relation between *pairs* of members of S . It is convenient at this point to introduce the mathematical construct of the *Cartesian product*, which is denoted by the notation $S \times S$. The Cartesian product is a set consisting of all possible ordered pairs (s_i, s_j) of the members of S . Thus, if $S = \{a, b, c\}$ then $S \times S = \{(a, a), (a, b), (a, c), (b, a), (b, b), (b, c), (c, a), (c, b), (c, c)\}$. The binary relation R is a subset, $R \subseteq S \times S$, and the members of R define the binary relation.

The binary relations we are interested in are ordering relations, and here we have two different types. They are called the **weak partial order** (also called a "poset") and the **strict partial order**. These binary relations are defined by specific properties concerning the members of R . A binary relation is said to have the **reflexive property** if for every $s \in S$ we have $(s, s) \in R$. Otherwise R is said to have the **irreflexive property**. R is said to have the **antisymmetric property** if for any pair of members, $a \in S \neq b \in S$, the inclusion $(a, b) \in R$ implies $(b, a) \notin R$. Finally, R is said to have the **transitive property** if for any three members a, b, c of S the inclusion of both (a, b) and (b, c) in R implies $(a, c) \in R$. A binary relation R is a weak partial ordering if R has the reflexive, antisymmetric, and transitive properties. R is a strict partial ordering if it has the irreflexive, antisymmetric, and transitive properties. It is a common symbolic convention to denote a weak partial ordering by the notation $a \leq b$ and to denote a strict partial ordering by $a < b$.

Suppose R is a weak partial ordering. Then $(s, s) \in R$ for every $s \in S$. Let I_A denote the set of all reflexive pairs (s, s) in R . Then the binary relation P defined by the set difference $P = R - I_A$ is a strict partial ordering. More generally, if $A \subset S \times S$ is any binary relation lacking the reflexive property and $I_A \subset S \times S$ is the set of all reflexive pairs in $S \times S$, the set $C = I_A \cup A$ is called the **reflexive closure** of A . If P is a strict partial ordering on S and R is the reflexive closure of P , then R is a weak partial ordering.

Next we must define what we mean by an **order structure**. A structure in general is a system of self-regulating transformations such that:

1. no new element engendered by their operations breaks down the boundaries of the system;
2. the transformations of the system do not involve elements from outside the system; and
3. the structure may contain within it differentiated substructures having their own local

transformations but necessarily having transformations from one subsystem to others such that the unity of the structure as a whole is conserved.

Once defined and laid down (as a closed system), a mathematical ordering qualifies as a structure under this definition and so it is proper and correct to call such a mathematical entity an order structure. However, the pure form of inner sense is not an order structure in this sense because such a presupposition is tantamount to regarding time as an innate and static "temporal template." Such a presupposition is neither justified in experience nor by any transcendental ground in Critical metaphysics. Nor is such a presupposition consistent with observable psychological facts evident in the empirical study of mental development in children. Rather, we must regard the pure intuition of time as a process of synthetic order *structuring* by which a temporal order structure is produced. **Structuring** is the act of putting into effect one or more transformations in a structure and in the case of temporal order structuring this means the construction of the form of inner sense as an **open system**.

Now, the transcendental-judicial Idea points us to the nature of the members of a set S from which the pure synthesis of time is structured. They will be *perceptions*, s . The transformations involved in the synthesis must, likewise, be transformations that both produce orderings and are such as to provide for the unconditioned unity in apperception mandated by Critical acroam. The binary relation, R , thus has for its members *pairs* of perceptions, $k = (s_i, s_j)$. We will call such a k a **kinetic**. Figure 3.3.1 illustrates this concept of a kinetic in time.

§ 3.3 Relation in the Synthesis of the Pure Intuition of Time

Modality adds nothing to the representation of the object as object. The connection of Modality is a connection between the representation and the Self of the Organized Being. In the case of a judgment, Modality is seen as a judgment of a judgment.

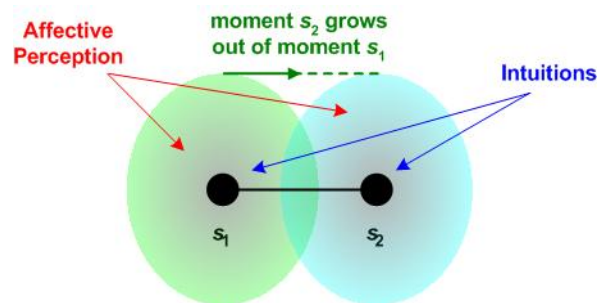


Figure 3.3.1: Basic kinetic of time. The kinetic includes both intuitions marked at moments in time as well as affective perceptions filling time. Moment s_2 is regarded as "growing out of" moment s_1 . Moments in time are "pulses of consciousness" denoting the representation (by reflective judgment) that a representation is presented in sensibility. Moments in time do not constitute time *per se*, which must be regarded as a continuum having no simple parts. Rather, moments in time represent the quality of consciousness in the *nexus* between the synthesis of apprehension and that of apperception.

To carry on with the theory of the synthesis of the pure intuition of time, we must now turn to the other three titles of representation to examine this *noumenal* object we call the pure intuition of time. Of these three titles, it is Relation that perhaps presents most clearly the character of subjective time and so we begin with it. The *modi* of persistence, succession, and coexistence speak to the properties attending the ordered structuring of time. In this regard, something Kant set down in *Critique of Pure Reason* has the utmost relevance for our discussion:

All which in our cognition belongs to intuition (excepting therefore the feeling of *Lust* and *Unlust* and will, which are not cognitions at all) contain nothing but mere relationships, of places in one intuition (extension), change of places (movement), and laws in accordance with which this change is determined (moving powers¹⁸). But what is present in that place, or what it brings about in the things themselves besides the change of place, is not given thereby. Now through mere relationships no Thing¹⁹ regarded as it is in itself is recognized; it is therefore right to judge that since nothing is given to us through outer sense except mere representations of relationships, this can also contain in its representation only the relationship of an object to the subject, and not that which reaches into the Object in itself. It is exactly the same in the case of inner sense. It is not merely that the representations of *outer sense* make up the proper stuff with which we occupy our mind, but moreover the time in which we set these representations, which itself precedes the consciousness of them in experience and, as formal conditions of the way in which we ground how we set them in the mind, already contain relationships of succession, of coexistence, and of that which is conjoint with succession (of persistence). Now that which, as representation, can precede any act of thinking is something in intuition and, if it contains nothing but relationships, it is the form of intuition, which, since it does not represent anything except insofar as something is set in the mind, can be nothing other than the way in which the mind is affected by its own activity, namely this setting of its representation, thus the way it is affected through itself, i.e. according to an inner sense of its form. [KANT1: B66-68]

The transcendental subjectivity of the pure intuition of time is made clear by Kant in this quote. The metaphysic proper of Rational Psychology provides the acroams that speak to what Kant called "the thinking nature" of the Organized Being, and from the judicial Standpoint the psychological Idea of Relation is: *Unconditioned unity of all relationships is grounded in the a priori anticipation of the form of connection of perceptions in time according to the modi of persistence, succession, and coexistence*. In company with this acroam is the logical-judicial acroam of Relation in Rational Physics: *Experience is possible only through the representation of a necessary connection of perceptions*.

The *modus* of succession in time is arguably the most commonly agreed upon feature of time. It is from this *modus* that the image of "time's arrow" is obtained. Figure 3.3.2 illustrates this.

¹⁸ The topological synthesis of space is a process that utilizes kinaesthetic feedback. It is thus reliant upon transformations of motoregulatory expression. It is this latter that constitutes the moving powers to which Kant refers in this quote.

¹⁹ *Sache*. This German word translates into English as "thing" but in a different connotation than the German word *Ding*. I therefore render *Sache* as "Thing" in translating Kant's words. A *Sache*-thing (Thing) is an object regarded from the empirical reflective perspective as a thing-in-the-world, the *Dasein* of which is a transcendental necessity for experience. But the necessity of this *Dasein* imputes nothing of *Existenz*.

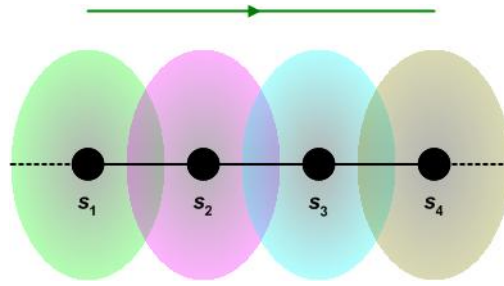


Figure 3.3.2: Succession in time depicted as a directed chain of successive moments in time.

Human experience yields up a common conception of time as having but one direction, normally described as proceeding from the past, to the fleeting "now" of the present, and onward into the future. Closer examination of this common manner of thinking about objective time does give rise to a number of fine philosophical issues, as discussed in Chapter 21 of *CPPM*, but what is key to our present purposes is simply this: **the transcendental aesthetic ground of the *modus* of succession in time is antisymmetric structuring in the making of perceptions in time.** The pure intuition of time has a "direction" in the manifold in sensibility and is irreversible insofar as the *modus* of succession in time is concerned.²⁰

Persistence in time is likewise an easily-grasped character of time, although in the case of this *modus* it is prudent to first point out what persistence in time is *not*. Persistence in time does not mean keeping any particular mental representation constantly in consciousness at every moment in time. Rather, persistence in time means the possibility of placing the representation of an object in intuition at any moment in time whatsoever. One does this, for example, when one thinks about "the-puppy-I-had-when-I-was-a-child." Assuming, for the sake of argument, that you are not now a child, that you did have a puppy when you were a child, and that this puppy long ago grew into a dog and perhaps has even died by now, you are still capable of imagining your long-ago puppy today. As an object, your puppy is "the same puppy now" as it was then, and the capability of being able to make a representation such as this one is all that persistence in time implicates.

Now, a specific Object *in concreto* is represented in Quantity as an identified object (general idea of identification in our 2LAR of representation-in-general) insofar as the *Dasein* of the object is thought. Temporal connections of inner sense, however, are always represented as a kinetic. For the *nexus* of persistence in time of a matter having the Quantity of identification, the

²⁰ This does not mean it is impossible to think of *objective* time as being able to "run backwards." Physicists think precisely this in quantum electrodynamics theory. This is discussed in *CPPM*, but here it should be enough to merely point out that objective time is an Object, that it is not the same thing as the pure intuition of subjective time, and that objective time is a *developed idea*. Infants and very young children, for example, have no innate concept of objective time (see [PIAG4, PIAG5]). Likewise, and for this same reason, it is possible to indulge in such fantasies as "time travel."

kinetic cannot be a kinetic of two distinct perceptions in time (because this is succession) but, instead, requires the possibility of the kinetic form $k = (s_i, s_i)$. Mathematically, this is nothing else than a reflexive relation and so **the transcendental aesthetic ground of the *modus* of persistence in time is reflexive structuring in the making of perceptions.** The synthesis of the pure form of inner sense constructs weak partial order structures. This aesthetical law of sensibility turns out to lead to a very interesting and unexpected consequence for the form of subjective time, as we will shortly see.

The *modus* of coexistence in time is quite probably the least obvious in terms of understanding our model of the pure intuition of time. If one examines again the previous two figures, it seems very natural to think something along the lines of "I see two perceptions, one affective and the other an intuition, represented at the same moment in time, so this illustrates what is meant by coexistence in time." However, there is a subtle error in thinking this. An intuition and an affective perception are representational *matters* and the synthesis of the pure intuition of inner sense does not itself deal with the matters of perception because time is only the *form* of inner sense. The correct and *transcendental* Logic we must use is the following. Referring to Figure 3.3.2 again, we note that this figure illustrates a chain of kinetics, i.e. (s_1, s_2) , (s_2, s_3) , etc. As an order structure, this chain directly implicates order relations $s_1 \leq s_2$ and $s_2 \leq s_3$. However, this conjunction in an order structure also implicates the order relation $s_1 \leq s_3$, and *this* connection is one that expresses in mathematical form the notion that something kinetically "shared" by s_1 and s_2 and something "shared" by s_2 and s_3 is *likewise shared* by s_1 and s_3 . Put into other words, there is something "in common" between the two kinetics such that $[s_1 \leq s_2 \text{ and } s_2 \leq s_3] \Rightarrow s_1 \leq s_3$. This is, of course, merely the transitive order relation whereby s_1 , s_2 and s_3 have a common *Existenz* in *one* subjective time. **The transcendental aesthetic ground of the *modus* of coexistence in time is transitive structuring in the making of perceptions.**

This is merely to say that the mathematical character of connections among kinetics in time is reciprocally determining. The transitive binary relation just stated is stated in a positive form, but there is a negative implication here as well. It is this: if $s_1 \leq s_2$ but $s_2 \not\leq s_3$ (that is, $s_2 \leq s_3$ is not contained in the partial ordering), then neither the ordering $s_1 \leq s_3$ nor $s_1 \not\leq s_3$ can be immediately concluded. Either predication is a logical *saltus*. However, if such partial orderings as $s_1 \leq s_2$ and $s_2 \not\leq s_3$ are *possible* in the pure form of inner sense, the acroam of unity in subjective time mandates that either $s_1 \leq s_3$ or $s_1 \not\leq s_3$ *must be determinable* (else s_1 , s_2 and s_3 could not be regarded as being in one and the same universal time).

The question before us, then, is: whether the situation-in-time $s_1 \leq s_2$ but $s_2 \not\leq s_3$ is a possible

form in inner sense. If we presume it is not then this is equivalent to presupposing that the form of inner sense is represented by what we commonly call a **time line**. But if we presume such a situation is possible in inner sense, *then the pure intuition of time cannot be merely a timeline*.

Which is it to be? Critical epistemology does not allow us to merely guess and presuppose an answer to this question; our model loses all grounds in objective validity if we do so. Objective validity is likewise lost if we dictate an answer *a priori* by some sort of Platonic fiat. To obtain the objectively valid answer we require, we turn to Quantity in the representation of the form of inner sense.

§ 3.4 Quantity in the Synthesis of the Pure Intuition of Time

The composition of Quantity in representation is a process of aggregation. The psychological Idea of Quantity in the judicial Standpoint is: Unconditioned functional unity of affective and objective perception in sensibility. This functional unity is judged by reflective judgment during the synthesis of apprehension and the Idea speaks to the co-importance of affectivity and cognition in the representations of sense. Alongside this Idea, we also have the acroam of the Axioms of Intuition in the judicial Standpoint: All intuitions are extensive magnitudes. To appreciate the significance of this Idea for the pure synthesis of time, we must examine what is meant by the term "extensive magnitude."

A **magnitude** is a determination of an object according to which the apprehension of its intuition is represented as possible only through the repeated positing of homogeneous parts. An **extensive magnitude** is a magnitude in which the representation of the parts precedes and makes possible the magnitude of the whole. It is this synthesis under the acroam of Axioms of Intuition that underlies *change* in sensibility, and this is at the root of the intuition of time. Kant tells us,

The pure image of all magnitudes (*quantorum*) for outer sense is space, but for all objects of the senses in general, time. The pure *scheme of magnitude* (*quantitatis*), however, regarded as a notion of understanding, is *number*, which is a representation integrating the successive addition of unit to (homogeneous) unit. Thus number is nothing other than the unity of the synthesis of a homogeneous intuition in general, because I beget time itself in the apprehension of the intuition. [KANT1: B182]

The basic unit of the partial ordering carried out in synthesis of the pure intuition of time is the ordered pair of perceptions (s_1, s_2). Now, each s_i in this kinetic includes the representation of an intuition, and the acroam of Quantity tells us that the two perceptions in a kinetic must differ from each other. Were this not so, there would be no ground for marking two distinct moments in time by reflective judgment. Indeed, **change** is perception of differentiable moments in time and is the most basic feature of the (mental) phenomenon of time. But by this very nature of extensive magnitude and differentiability in time, the kinetic (s_1, s_2) is *irreflexive*, i.e. $s_1 < s_2$.

If the pure intuition of inner sense took on merely the form of a time line, reflexive partial ordering, i.e. $s_1 \leq s_1$, would not be possible because this would be in violation of the judicial Axioms of Intuition across. However, as we have already seen, the *modus* of persistence in time requires that the order structure erected by the synthesis of time be a reflexive order structure. Because the form of a time line permits only strict partial ordering under the Critical across, the conclusion follows: *the form of the pure intuition of time cannot be composed only as a time line*. The immediate corollary of this is: **time takes the form of a multi-dimensional timescape**. Figure 3.3.3 below illustrates this concept of subjective time.

A number of psychological consequences follow from this surprising and rather unexpected mathematical character of the pure intuition of time. Let S be the set in a partial ordering relation and let x and y be two members of this set. Along any particular time line, x and y are said to be **comparable** if one or the other of the irreflexive orderings $x < y$ or $y < x$ appear in the time line. If every x and y in S are comparable then the partial ordering is said to form a total order or **chain**.

Now, in the illustration of Figure 3.3.3, the members $\{s_i, s_k, s_m\}$ are comparable, as are the members $\{s_i, s_j, s_l, s_m\}$. However, the pairs $\{s_j, s_k\}$ and $\{s_k, s_l\}$ are *not* comparable because these perceptions fall on different time lines within the timescape. Among other things, the *Existenz* of non-comparable perceptions in time provides a straightforward explanation for certain types of psychological neuroses, such as those until recently called hysterical neurosis (see Chapter 22 of *CPPM* for a discussion of this). In the terminology of mathematics, perception s_i is called a **meet** for the two time lines diverging from it. Perception s_m is similarly called the **join** of the time lines converging at it.

The generation of perception s_m differs from those of all other moments in time in the submanifold depicted in Figure 3.3.3. In any kinetic (s_1, s_2) , s_1 is called the **direct cover** of s_2 . In this figure, s_m differs from other perceptions in the submanifold by virtue of its having *two* direct covers (s_k and s_l). This is because s_m is where two time lines join. Here again we must account for

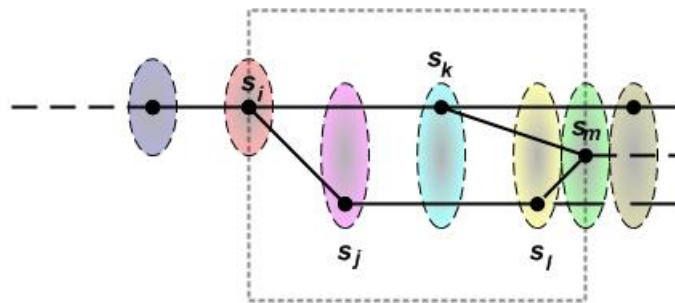


Figure 3.3.3: Illustration of a timescape. Black dots denote intuitions, colored ovals represent affective perceptions spanning the multiple timelines contained in the timescape. The dotted box denotes a submanifold in time bounded by s_i and s_m .

the laws of the Critical acroams in considering the generation of s_m . Although allowance must be made for new effects of receptivity and spontaneity in the synthesis of apprehension (possible new *materia in qua* for the intuition at s_m that were not present at s_i), it is nonetheless the case that the intuition of s_m necessarily involves some contribution to its contents common to both s_k and s_l . This is because s_m must survive the *Verstandes-Actus* of abstraction and unless it contains *materia in qua* common to both its covers the intuition of s_m is empty (devoid of matter). But an empty intuition is *nothing* – not an object of a possible objective perception. Furthermore, since the two cover intuitions are non-comparable, what is common in both must originate at s_i , the meet of the two time lines in the submanifold.

Thus, here in the generation of the join s_m we have the ground of possibility for representation of something *persistent in time*, i.e. for a reflexive ordering (s_m, s_m). In abstract mathematics, e.g. graph theory, the reflexive relation is typically illustrated by a self-loop (a graph vertex with an edge running from that vertex immediately back to itself as its own direct cover). However, this sort of graph-theoretic structure is not objectively valid (indeed, it is objectively *invalid*) for the pure intuition of time because the idea of a moment in time re-generating itself in one and the same moment in time is self-contradictory.

By itself, this reflexive order relation made possible by joins in the timescape does not implicate reflexive order relations for the other perceptions between the meet and join points. However, this is not the case when the synthesis of apprehension involves the re-production of *materia in qua* originating in the spontaneity of reproductive imagination from concepts. This is because, as we will later see, concepts are constructed in accord with pure notions of understanding (the categories of understanding), and these notions preserve the form of inner sense in re-presentation of the concept. (The synthesis of apprehension in this case is called a synthesis of **comprehension**). Concepts re-introduced into sensibility by reproductive imagination carry with them the time-determinations of the original intuitions that gave rise to the concepts in the synthesis of re-cognition in imagination (refer to Figure 1.5.1). In the multi-dimensional timescape of inner sense, these time-determinations in concepts introduce further possibilities for joins involving the *materia* of concept reproduction. However, the *original possibility* for a time-determination of persistence lies with the apprehensive join illustrated in Figure 3.3.3.

§ 3.5 Quality in the Synthesis of the Pure Intuition of Time

The form terms (Relation and Quantity) in the 2LAR of the pure synthesis of time are relatively easy to describe. The matter terms (Modality and Quality) are less so. In its logical

essence, the pure intuition of time is a form-building process and its character is mathematical. Moments in time are not part of time proper but are merely markers resulting from acts of reflective judgment. In what, then, do we find the matter in composition for the synthesis of the pure intuition of time?

The logical-judicial Idea of Quality in Rational Physics states: The intensive magnitude (degree) of sensation presents the complete condition for marking sensibility at a moment in time. The transcendental-judicial psychological Idea of Quality states that the division between objective and affective perception is a merely logical division, and that affective and objective perception in combination make up the complete state of conscious representation. (This is the Idea of unconditioned unity in the compatibility of representations). Both of these Ideas speak not to time *per se* but rather to what is said to "fill time" (provide matter for the form that time provides).

Referring once more to Figure 3.3.3, the affective *materia* of perception overlays the structuring of the timescape like a quilt blanket. Affective perception is precisely that in perception which is no part of any cognition and yet is nonetheless part of conscious representation. In his *Prolegomena to any Future Metaphysics*, Kant wrote:

The above-mentioned first principles²¹ are not referred directly to appearances of their relationship, but to the possibility of experience, for which appearances constitute only the matter but not the form, i.e. they are referred to the objectively and universally valid synthetic propositions through which judgments of experience²² are distinguished from mere judgments of perceptions²³. This happens because: appearances, as mere intuitions *that fill part of space and time*, are subject to the concept of magnitude, which synthetically unifies the multiplicity of intuitions *a priori* according to rules; and because the real in appearance must have a degree insofar as perception contains, beyond intuition, sensation as well, between which and nothing . . . a transition always occurs by diminution . . . but yet the transition to sensation from empty time and space is possible only in time with the consequence that although sensations . . . can never be known *a priori*, it²⁴ nonetheless can be distinguished intensively from every other of the same kind; from which, then, the application of mathematics to nature with respect to sensuous intuition . . . is first made possible and determined. [KANT: 4 (308-309)]

Intensive magnitude is defined as a unity (of magnitude) in which the idea of multiplicity can be represented only by an approximation to negation. In contrast to extensive magnitude, intensive magnitude is presented in consciousness all at once and as a whole (whereas extensive magnitude is 'built up' part by part). This character that the singular whole precedes any representation of any parts within it – and thus is perceived as a 'unit' – distinguishes intensive from extensive magnitude. The concept that this presentation contains a multiplicity within it (and is therefore a

²¹ of Rational Physics

²² judgments of determining judgment

²³ judgments of reflective judgment

²⁴ the real in appearance, i.e. the transcendental object

magnitude) is made possible only from the fact of experience that sensations seem to grow or diminish qualitatively without in any other way being perceptibly different. As Bergson jovially remarked one time²⁵, "It seems evident that we experience a more intense pain at the pulling out of a tooth than of a hair." This growth or diminution appears without any perceptible unit of measure, i.e. is sensibly continuous. There is no "simple part" of which the magnitude can be said to be composed by aggregation. This situation is quite comparable to the difference between the cardinal numbers (where '1' is the fundamental unit and succeeding numbers are built by adding '1' to their predecessors) and real numbers (for which there is no smallest real number and which are understood only through the imaginative process of dividing up the number line into sets; mathematicians call this process "making a Dedekind cut")²⁶. So-called "infinitesimal real numbers" are defined through this same sort of limiting process and the inductive procedure mathematicians use (illustrated in Figure 3.3.4) to "zero in" on a "point on the number line" is a good metaphor for this Critical notion of diminution of sensation. Note that this mathematical procedure (which is never completed in actuality; mathematicians propose only the formal possibility of completing it, "completion" being *defined* by Weierstrass' definition of "mathematical continuity") is nothing else than a process of successive strict partial orderings.

Because an intensive magnitude is presented as a 'unit' of representation at any particular moment in time, and yet sensations presented at two different moments in time can be compared (in the connotation that one can be perceived to be "more" than the other), it is only through ordering in the structure of time that intensive magnitude can be known to *be* a magnitude at all. The 'matter conveyed in time' is *change*, and thus change can be called the information contained in a kinetic. Time *per se* is not sensuous but that which is conveyed to consciousness by the act of reflective judgment *is* sensuous.

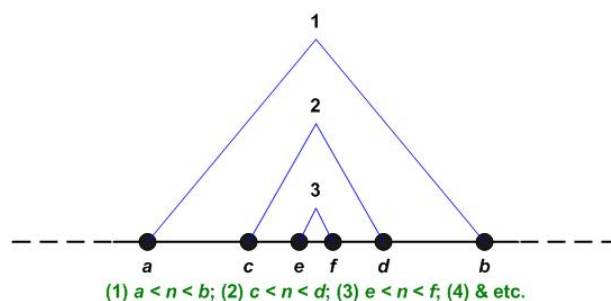


Figure 3.3.4: Ordinal procedure for inductive definition of a real number, n .

²⁵ Henri Bergson, *An Essay on the Immediate Data of Consciousness*, 1889.

²⁶ More formally, let the number line be divided into two disjoint sets, A and B , such that for every number $a \in A$ and every number $b \in B$ we have $a < b$ (a strict partial ordering). The mathematical point at which the number line is cut up into the two sets (A , B) is regarded as the real number defined by the cut. The set of all real numbers is defined as the set of all Dedekind cuts. Numbers so defined are secondary quantities.

Quality is the matter of the matter of representation (the matter in a composition) and at root pertains to "what is composed" (Quantity is "how it is composed") in the act of representation. The Quality of time synthesis is exhibited by its capacity to make it possible (through affective perception) for reflective judgment to affirm *materia in qua* and negate *materia ex qua* in sensibility. In this basic context, the Quality of the pure form of inner sense subsists in the relationship of affective perception to reflective judgment and, through this, to the sensorimotor capacity of *psyche*. We will see later in this book that informative change delimited by the kinetic of time grounds the possibility of what we will call the *motivational dynamic* of judgmentation.

§ 4. The Aesthetic of the Acts of Understanding

Reflective judgment is the determining factor at every step in the three-fold synthesis of the *Verstandes-Actus* of sensibility. The functional character of *Comparison*, of reflexion, and of abstraction follows from the manner in which reflective judgment adjudicates the possibilities provided in the obscure representations in sensibility during the synthesis of apprehension. The deduction of these characteristics for the acts of understanding is provided in Chapter 14 of *CPPM*, and this deduction is not brief. It involves considerations dealing with the *Realdefinition* of the categories of understanding in determinate judgments, the functional *momenta* of aesthetical reflective judgment, and with the applied metaphysic of *psyche* (called the sensorimotor idea). We may note that these three factors are those illustrated in Figure 3.1.1 as the processes enveloping the synthesis of apprehension in sensibility. In this book we will proceed directly to the conclusions reached in *CPPM* and merely explain the acts themselves.

§ 4.1 *Comparison and Association*

Association is Critically defined as: (1) the function of aesthetic Quantity producing a relationship of commonality for two or more representations in conscious presentations; (2) the aggregation so formed. The representations in an association have commonality in an interest, desire, purpose, or as matters of an act. Their association (by means of an act of synthesis) is the unity of this commonality. The ideas of interest, desire, etc. are ideas rooted in affectivity and ultimately grounded in the *momenta* of aesthetical reflective judgment. Reflective judgment does not deal in objective representations and acts of reflective judgment deal in general merely with the feeling of *Lust* or *Unlust* in affective perception. Aesthetic Quantity, as form of composition, must always be regarded in terms of combination joining sensibility with the Organized Being's *state of mind*. The technical terms for this are ***Wohlgefallen*** (satisfaction) and ***Mißfallen*** (dissatisfaction).

Wohlgefallen is a sense of satisfaction describable as "this is *not-bad*." Similarly, *Mißfallen* is

a sense of dissatisfaction describable as "this is *not-good*." *Wohlgefallen* denotes a particular sense of complacency while *Mißfallen* is a sense of disturbance or ill-being. Something to note about these two terms is the curiously negative character of their connotations, i.e., "not-bad" and "not-good." Reflective judgment knows no objects and is governed and regulated by the Critical acroam of the principle of formal expedience for a purpose of pure practical Reason. It can, at root, only make judgments of expedience ("not-bad") or in expedience ("not-good") in regard to determining the state of mind of the Organized Being. In relationship to sensibility,

The subjective representation of the collective power of life to receive or exclude objects is the relationship of satisfaction or dissatisfaction. Thus the feeling is not the relationship of the object to the representation but rather to the collective power of mind, either to most intimately receive or to exclude the same. [KANT: 28 (247)]

Satisfaction-dissatisfaction, as that in affective perception which serves to "receive or exclude" representations (a function that speaks to identification, differentiation, and integration, i.e., Quantity) serves as a *criterion for comparison* in judging formal expedience. To realize this function requires an *act* of mind, and so we have: ***Comparison is the act in the synthesis of apprehension by which representations are either associated or not-associated in affective perception through a sense of satisfaction or dissatisfaction.*** This is the Critical explanation of what it is "to liken representations" in apprehension. *Comparison* is the act of synthesis and association is the commonality-producing function of aesthetical representation in Quantity.

§ 4.2 Reflexion and Compatibility

Compatibility is the coalescing function for comparates in aesthetic Quality, by which reflexion is referred to the Organized Being's faculty²⁷ of knowledge. Aesthetic Quantity pertains to form of composition of a state of satisfaction-dissatisfaction; aesthetic Quality pertains to the matter of this composition. We call the sensational matter of an affective perception a ***feeling in the narrow sense.***²⁸

[The] subjective in a representation *which cannot become part of cognition at all* is the feeling of *Lust* or *Unlust* combined with it; for through this I know nothing about the object of the representation, although it can well be the effect of some cognition. Now the expedience of a thing, so far as it is represented in a perception, is not also characteristic of the Object itself (for such a thing cannot be perceived), although it can be deduced from a cognition of things. Thus the expedience that precedes the cognition of an Object – which moreover is immediately combined with it without wanting to use the

²⁷ The Critical term "faculty" must not be confused with the old and now discredited "faculty psychology" movement of the eighteenth and nineteenth centuries. A **faculty** is the form of an ability insofar as the ability is represented as an idea of organization. The **faculty of knowledge** is the systematic structure of the ability to make representations of knowledge.

²⁸ A feeling in the wide sense is a designation denoting a Quality or a Modality in an aesthetical reflective judgment.

representation of it for a cognition – is the subjective that cannot become part of cognition at all. The object is therefore called expedient in this case only because its representation is immediately combined with the feeling of *Lust*; and this representation itself is an aesthetic representation of expedience. [KANT: 5 (189)]

The function of compatibility coalesces comparates in sensibility (*materia ex qua*) according to the principle of formal expedience, and here the question that faces us is: In what manner is this coalescence of feeling expedient for a purpose of Reason? We cannot state this in terms of any objective purpose because reflective judgment knows no objects. The expedience must, rather, refer to something else. But, aside from cognition, all that is left to us here is *subjective* expedience in the congruence or "harmony" of the capacity of the Organized Being to form sensuous representations of knowledge in the *wide* sense. **Knowledge in the wide sense** is any conscious representation or capacity for making such a representation by or through which meanings are determined. But *subjective* meanings can only be understood in terms of actions undertaken by the Organized Being (acting as agent). Put in other words, the *subjective and practical meaning* of something is *what can be or is done with it*. Therefore, expedience in this context refers to the congruence in the actions of the logical divisions of *nous* (sensibility, reflective judgment, and determining judgment) in the act of making representations conscious. Because these divisions of *nous* are merely logical divisions, our theory must regard them as reciprocally-determining (principle of emergent properties) and this reciprocal co-determination is what is meant by the "harmony" of these processes.

Compatibility is thus seen as the "harmony-producing function" of the synthesis of apprehension. The place where the noetic processes meet in representation and judgmentation is, of course, sensibility, and here the question we must ask is: What exactly is it that is harmonized in the synthesis of apprehension? To understand this, let us look at the illustration of noetic organization depicted in Figure 3.4.1.

We call this figure *the cycle of thought*. **Thinking** is cognition through concepts, and here the most immediate capacities involved in the making of cognitions are those of the synthesis of apprehension, the synthesis of imagination (both in re-cognition and reproduction), and the process of determining judgment. We call this the *inner loop* in the cycle of thought. However, the possibility of thinking also calls upon the other capacities of reflective judgment (in adjudicating the representations of sensibility) and the *outer loop* from sensibility, through reflective judgment, to the processes of Reason, to determining judgment, and back to sensibility once more via the synthesis of imagination in reproduction. The operation of this outer loop is **judgmentation** in general (*Beurtheilung* in Kant's terminology).

We have, then, three possibilities for the manner in which harmonization of noetic acts can be

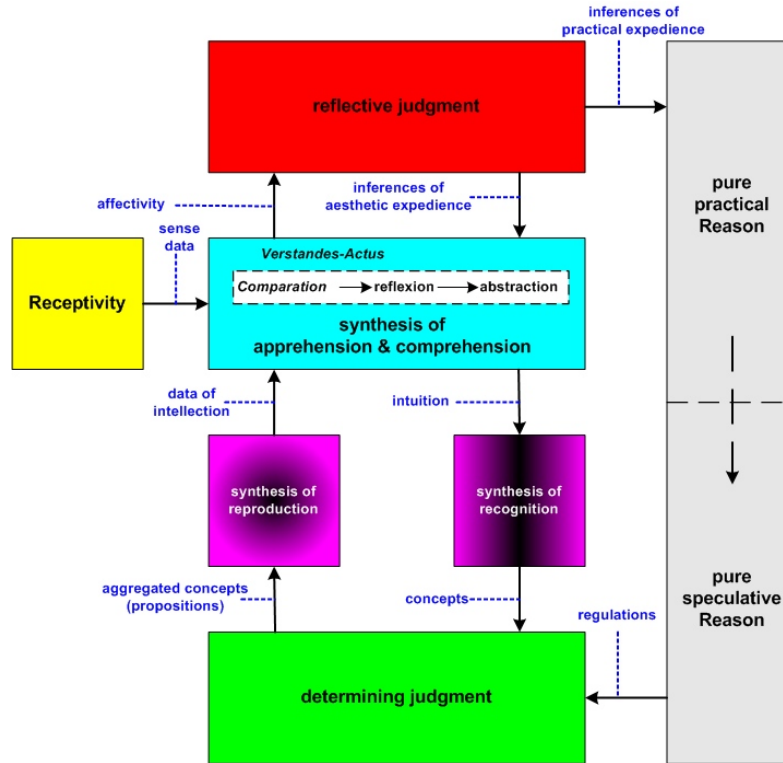


Figure 3.4.1: The cycle of thought. Thinking is cognition through concepts, but the possibility of thinking calls upon more than merely the synthesis of apprehension, determining judgment, and imagination. All the logical divisions of *nous* represented in this diagram play a necessary role in making thinking possible. accomplished.²⁹ The first is harmonization of the inner loop, the action of which we call the **free play of imagination and understanding**. The second is harmonization in the judicial loop of sensibility and reflective judgment, and this we can call aesthetic harmony. The third is harmonization of the outer loop of judgmentation which, because it involves acts of practical and of speculative Reason, we call the harmonization of reasoning.

The *realization* of any one of these possible harmonies is an expedient outcome for the cycle of thought. **Reflexion is the act of coalescence in sensibility that produces a representation harmonizing any one of these three loops.** In a slightly metaphorical sense, because all three of these loops "meet" in the synthesis of apprehension the act of reflexion can be said to determine the [transcendental place](#) where co-determined compatibility of the noetic processes is achieved. Reflexion is the act in the synthesis of apprehension for which affective compatibility in the Quality of sensuous representation is the function.

§ 4.3 Abstraction and Transcendental Anticipation

We use the word **anticipation** to mean [knowledge through which the Organized Being can](#)

²⁹ **Harmonization** is the act of making diverse representations compatible and homogeneous with one another so that they may be combined in composition.

know and determine *a priori* what belongs to empirical cognition. A **transcendental anticipation** is anticipation in the form of knowledge *a priori* that is necessary for the possibility of making perceptions through the synthesis of reproduction in imagination. The acroam for all transcendental anticipations is the Idea that unconditioned unity of all relationships is grounded in the *a priori* anticipation of the form of connection of perceptions in time according to the *modi* of persistence, succession, and coexistence, which is none other than the psychological Idea of Relation from the judicial Standpoint. The *function* of anticipation in the combination of representations has (because this function is *a priori*) the forward-looking character required for the determination of a *practical* representation (by pure practical Reason) of a determined practical purpose. (This type of representation is called an *appetite*)³⁰.

Abstraction is the act of removal or exclusion from perception of inexpedient *materia* from the final determinations of sensibility at a moment in time. It is this character of the act of abstraction that makes possible the synthesis of higher concepts by providing the intuitions that will become these higher concepts following the synthesis of recognition in imagination. Abstraction makes possible a connection (*nexus*) of *subjective* judgment (by reflective judgment) and is not limited to merely the representation of intuitions. The act of abstraction is the actualization of Attention (the act whereby a particular representation is made clear and conscious while others are kept unconscious, i.e. obscure). Thus the act of abstraction and the function of transcendental anticipation both pertain to connections of Relation in a manifold. For abstraction this is Relation in the manifold of sensibility; for transcendental anticipation it is Relation in what we will be calling the manifold of Desires in reflective judgment. The act of abstraction and the act of making a reflective judgment (insofar as the function of transcendental anticipation in this act is concerned) are co-determining acts in the free play of the judicial loop between reflective judgment and the synthesis of apprehension.

§ 4.4 Summary

The *Verstandes-Actus* are acts of synthesis for sensuous representation. These acts are intimately linked to functions of representation in the process of reflective judgment. It is important for us to note the distinction between an act and a function. An act is the making of a *nexus* in an organized manifold of representations. A function is the unity of the act of organizing different representations under a common one. The *Verstandes-Actus* and the acts of reflective

³⁰ We will draw an important distinction between the terms appetite (in German, *Begierde*), desire (*Begehren*), and desiration (*Begehrung*). The latter two are matter and form, respectively, of affective perceptions in reflective judgment. An appetite, as a representation of pure practical Reason, is not a perception at all.

judgment are co-determining in the free play of the judicial loop (Figure 3.4.1) and there is a representational homogeneity between the acts of understanding and the judicial functions. We can illustrate this homogeneity in tabular form as:

<u>Act</u>		<u>Function</u>		<u>Title</u>
<i>Comparison</i>	$\Leftrightarrow$	Association	$\triangleleft$	Quantity
Reflexion	$\Leftrightarrow$	Compatibility	$\triangleleft$	Quality
<u>Abstraction</u>	$\Leftrightarrow$	<u>Transcendental anticipation</u>	$\triangleleft$	<u>Relation</u>

The synthesis of apprehension does not carry a title of Modality for the *Verstandes-Actus* because the acts of understanding are acts of synthesis and not titles of representation. Modality, as the matter of *nexus*, is the title that speaks to apperception, and in the synthesis of sensibility it is reflective judgment that stands as the determining factor (Modality of the synthesis) because sensibility does not judge. The titles of Quantity, Quality, and Relation attach to the *Verstandes-Actus* only because of the relationship of these acts to the functions in reflective judgment, i.e. the titles of Quantity, Quality, and Relation are the *contexts* for the acts of understanding.